MPEG Traffic Generation Matching Intra- and Inter-GoP Correlation

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Abstract
VBR compressed video traffic is expected to constitute the majority of the B-ISDN network load in the immediate future. At the moment, the MPEG coding scheme is widely used for many types of applications. On account of the high burstiness and periodicity of the traffic generated by an MPEG encoder, it is crucial to obtain a model for the generation of sequences with the same statistical characteristics of real MPEG traces. In this paper, after demonstrating that intra-GOP correlation, that is, the correlation between frames belonging to the same GoP, has to be taken into account, an accurate model allowing the generation of an MPEG-like trace is proposed. Moreover, as the MPEG traffic has been demonstrated to be self-similar, that is, it presents a long-range dependence (LRD), the proposed algorithm applies a method based on the Fast Fourier Transform (FFT) in order to generate self-similar traces. This algorithm allows to generate a trace, imposing the SRD and LRD behavior and the probability density function of any given video sequence.

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1. Introduction

The main characteristic of B-ISDN networks, achieved by using the Asynchronous Transfer Mode (ATM), is to support Variable Bit Rate (VBR) connections, so as to exploit the available bandwidth efficiently. Thanks to this property of B-ISDN networks and to the ever growing diffusion of multimedia applications in a distributed environment, VBR compressed video traffic is expected to constitute the majority of the network load in the immediate future.

At the moment, the MPEG coding scheme is widely used for many types of applications. Although there are actually two consolidated MPEG schemes [1-2]. On account of the high burstiness and periodicity of the traffic generated by an MPEG encoder, there are open issues concerning transmission on high-speed networks, like the dimensioning of multiplexer buffers, the choice of a good Call Admission Control (CAC) policy, monitoring and policing the network traffic, etc. Therefore, it is crucial to obtain an analytical model for the generation of sequences matching the statistical characteristics of real MPEG traces.

In literature we can find a very large number of models for non-MPEG video sources [3-6]. The majority of these are Markov-based [3-5], and therefore not adaptable to the MPEG traffic which, as demonstrated in [7], is self-similar, that is, it presents a long-range dependence (LRD), a characteristic that the classical, Markov-based models, like the Markov modulated Poisson processes (MMPP) [8] and the switched batch Bernoulli processes (SBBP) [9], are not able to capture. A first step in the analysis and modeling of MPEG video traffic was taken in [10], where a model based on a Gamma/Pareto distribution was proposed. This model is able to capture two main statistical characteristics of a real video sequence: the marginal distribution with a heavy tail, and the long-range part of the autocorrelation function (ACf). As this model is not very accurate in approximating the short-range dependence (SRD), it is only good when it is used to model an aggregate of a high number of video sources because, if that is the case, the marginal distributions become more Gaussian and the special short-range time correlation effects are randomized.

Another model for traffic emitted by MPEG video sources was proposed in [11]. This model allows a synthetic trace with both short- and long-range autocorrelation structures to be generated. In particular, the proposed algorithm generates the I-, P- and B- subsequences independently, applying the three different probability density functions (pdfs) of the three empirical I-, P-, and B- sequences to a single stationary background process. However, neither of
the above models takes into account the correlation between frames belonging to the same Group of Pictures (GoP).

In this paper we will focus on this correlation and then propose a more accurate model allowing the generation of an MPEG-like trace, taking into account intra-GoP correlation and, in particular, the correlation between P- and B- frames and the I-frame of the same GoP. The algorithm is based on the generation of a self-similar process for the I-frames; then, from these, the P- and B- frames belonging to the same GoP are generated. As regards the generation of a self-similar process with pdf and ACf imposed, a valid algorithm is available in [11], but its core is not very efficient because it is based on the Hosking’s procedure [12] and, therefore, it requires a very long computation time. In this paper a new algorithm is proposed, based on the Fast Fourier Transform (FFT) method, already used in [13] to generate a self-similar sequence, but without managing to impose the SRD behavior and the pdf.

This paper is organized as follows. Section 2 briefly defines self-similar processes, illustrates some of their properties, and describes the procedures to calculate the Hurst parameter, $H$. Section 3 describes the MPEG coding technique and its implications for traffic statistics, in terms of ACf, pdf, Hurst parameter, mean and variance; in Section 4 we will describe the new algorithm, based on the FFT method, to generate a self-similar trace with a given set of statistical parameters and we will use this procedure in the algorithm for the generation of an MPEG-like trace. Finally, in Section 5, we will evaluate the proposed algorithms and, in Section 6, provide our conclusions.

2. **Self-similar processes**

In this section we define self-similar processes and describe their main properties. It is only a brief outline and, for a more detailed presentation, the reader is referred to overview papers on self-similarity, like [14-17] and the references therein.

2.1 **Definition and properties**

Let $Y(n)$ be a discrete-time arrival process. Let us define as “Timescale of order $m$” for the process $Y(n)$ the timescale obtained from the original timescale, in which the Time Unit (TU) is the time slot, by aggregating non-overlapping blocks of $m$ slots. In this timescale the Time Unit is $TU = m$ slots. For each $m = 1, 2, \ldots$, let $Y^{(m)}(n)$ be the process in this timescale, obtained by
averaging the values of the original process $Y(n)$ over non-overlapping blocks of size $m$. $Y^{(m)}(n)$ is therefore the aggregate process defined as follows:

$$
Y^{(m)}(n) = \frac{1}{m} \left[ Y(n \cdot m - m) + Y(n \cdot m - m + 1) + \ldots + Y(n \cdot m - 1) \right], \quad n \geq 1
$$

(1)

The process $Y(n)$ is called **exactly second-order self-similar** with a Hurst parameter $H = 1 - \beta/2$ if, for all $m = 1, 2, \ldots$, the variance and the autocorrelation function of the aggregate process $Y^{(m)}(n)$ are:

$$
\text{Var}\{Y^{(m)}\} = \text{Var}\{Y\} \cdot m^{-\beta} = \text{Var}\{Y\} \cdot m^{2H-2}
$$

(2)

and

$$
R_{Y^{(m)}Y^{(m)}}(h) = \frac{1}{2} \left[ (h+1)^{2H} - 2h^{2H} + |h-1|^{2H} \right], \quad h \geq 0
$$

(3)

where $R_{Y^{(m)}Y^{(m)}}(h) = E \{ Y(n) \cdot Y(n + h) \}$ represents the autocorrelation function of the process $Y(n)$.

The process $Y(n)$ is called **asymptotically second-order self-similar** with a Hurst parameter $H = 1 - \beta/2$ if, for all $h$ large enough, we have:

$$
R_{Y^{(m)}Y^{(m)}}(h) \rightarrow R_{Y^{(m)}Y^{(m)}}(h), \quad \text{as} \quad m \rightarrow \infty
$$

(4)

In other words, $Y(n)$ is exactly second-order self-similar if the corresponding aggregate processes $Y^{(m)}(n)$ show the same variability as the process $Y(n)$ in any timescale; $Y(n)$ is asymptotically second-order self-similar if the corresponding aggregate processes $Y^{(m)}(n)$ show a variability which is more similar to that of $Y(n)$ as the timescale is larger.

The dependence of the variability of any arrival process on the timescale can be evaluated by measuring the variation of $\text{Var}\{Y^{(m)}\}$, when the timescale changes [17]. Thus, since

$$
\text{Var}\{Y^{(m)}\} \propto m^{2H-2}
$$

(5)

and

$$
\begin{cases}
0.5 < H < 1 & \text{for self - similar processes} \\
H = 0.5 & \text{for Markov - based processes}
\end{cases}
$$

(6)

the variability of Markov-based processes, and therefore their autocorrelation function, decreases more rapidly than that of the measured traffic, when the aggregation order $m$ tends to infinity.
2.2 Hurst parameter estimation

The estimation of the Hurst parameter of a sequence with a finite number of samples is an open problem because, by its nature, it should be calculated considering an infinite number of samples. In literature many methods for the evaluation of \( H \) have been proposed [17], some more accurate or fast than others. In this subsection we illustrate two of these methods, the most simple ones: Variance-Time plots and R/S analysis.

The first algorithm is based on (2), which is used to calculate the variance of the aggregate process \( Y^{(m)}(n) \) as a function of the variance of the process \( Y(n) \) and of the aggregation level \( m \).

Thus, by using (2) and plotting the function \( \log(\text{Var}(Y^{(m)})) \) against \( \log(m) \), an estimation of \( \beta = 2 - 2H \) is the absolute value of the asymptotic slope of the simple least squares line fitting the resulting points in the plane, ignoring the small values of \( m \).

The R/S analysis method is based on the Hurst effect: for a given set of observations \( \{Y_k\} \), \( k \in [1,\ldots,n] \), with sample mean \( \mu_y(n) \) and sample variance \( S_Y^2(n) \), we have the following relation [18]:

\[
E\left\{R(n)/S(n)\right\} \propto n^H \quad \text{as} \quad n \to \infty
\]

(7)

where

\[
R(n)/S(n) = \frac{1}{S(n) \cdot \left[\max(0, W_{i_1}, \ldots, W_{i_K}) - \min(0, W_{i_1}, \ldots, W_{i_K})\right]}
\]

(8)

is the rescaled adjusted range or “R/S statistic” and \( W_k = (Y_i + Y_{i+1} + \ldots + Y_{i+k}) - k \mu_y(n), k \in [1,\ldots,n] \).

Given a sample of \( N \) observations, \( \{Y_k : k \in [1,\ldots,N]\} \), one subdivides the whole sample into \( K \) non-overlapping blocks and computes the rescaled adjusted range \( R(i,n)/S(i,n) \) in each interval \( C_i = [(i-1)N/K, iN/K] \), for each \( i \in [1,\ldots,K] \). Here the R/S statistic \( R(i,n)/S(i,n) \) is defined as in (8) with \( W_k \) replaced by \( W^{(i)}_k = \sum_{j \in C_i} Y_j \), and \( S^2(i,n) \) being the sample variance of \( \{Y_k\} \), with \( k \in C_i \). Next we plot the function \( \log\left(R(i,n)/S(i,n)\right) \) versus \( \log(n) \). This plot is the rescaled adjusted range plot (also called the “Pox diagram of R/S”). An estimate \( H \) is given by a least squares fit.
3. MPEG peculiarities

In this section we present the characteristics of the MPEG-1 coding technique. In particular, in Subsection 3.1 we describe the structure of the traffic generated by an MPEG encoder and, in Subsection 3.2, we discuss the statistical properties of an MPEG data flow. These properties will be used in Section 4 in the generation algorithm for MPEG video sources.

3.1 MPEG coding technique

The basic idea behind MPEG video compression is to remove spatial redundancy within a video frame and temporal redundancy between video frames. The MPEG video algorithm can compress video signals to an average of about ½ to 1 bit per coded pixel. For example, when a compressed data rate of 1.2 Mbits per second, and a coding resolution of 352×240 at 30 Hz are used, the resulting video quality is comparable to VHS. The MPEG-encoder input is a sequence of video frames. Each frame is a still image. A video player displays one frame after another, usually at a rate close to 30 frames per second (23.976, 24, 25, 29.97, 30). A frame is a bidimensional array, with a certain number of rows, each of which has a number of picture elements (pels), according to the national video standard adopted (NTSC, PAL, etc.). The encoder output is a deterministic periodic sequence, in which the period is a Group of Pictures (GoP) realized with three types of encoded frames:

- **I frames** are coded using only information present in the picture itself, in order to provide potential random access points into the compressed video sequence. The coding is based on the discrete-cosine transform according to the JPEG coding technique [19];
- **P frames** are coded using a similar coding algorithm to I-frames, but with the addition of motion compensation with respect to the previous I- or P-frame (forward prediction);
- **B frames** are coded with motion compensation with respect to the previous I- or P- frame, the next I- or P- frame, or an interpolation between them (bidirectional prediction).

Typically, I- frames require more bits than P- frames, and B- frames have the lowest bandwidth requirement. The GoP structure, as well as the number of frames composing it, is not established in the standard. A widely used GoP structure is constituted by 12 frames, alternated as follows: IBBPBBBPBBPBB. The coding relationships among the frames in this particular case are shown in Fig. 1.
3.2 MPEG statistical properties

For the sake of conciseness, in this paper we present only the results obtained for the frame-size sequence of the coded movie “The Simpsons”, even if the considerations and the algorithm proposed here are valid for all the sequences we have tested, a number of which loaded in the ftp site ftp-info3.informatik.uni-wuerzburg.de/pub/MPEG, other generated from original movies. This sequence has already been studied in [20], where distributions and correlation functions are evaluated. What we add in this paper, and then we use in the algorithm in Section 4, is the intra-GoP correlation, that is, the correlation between frames belonging to the same GoP.

We will indicate the frame-size sequence, expressed in bits, of the whole movie with the term “movie trace”, and the I-, P- and B- frame-size sequences with the term “subtraces”. In Tables 1-4 the statistical parameters, mean and variance of each frame in the GoP are listed. As can be seen, we have:

\[
\mu_{P_i} \equiv \mu_{P_j} \equiv \mu_{P_k} \equiv \mu_{P} = 23.9 \cdot 10^3 \text{ [bit / frame]} \tag{9}
\]

\[
\sigma^2_{P_i} \equiv \sigma^2_{P_j} \equiv \sigma^2_{P_k} \equiv \sigma^2_{P} = 297 \cdot 10^6 \text{ [bit}^2 / \text{frame}^2]\tag{10}
\]

\[
\mu_{B_i} \equiv \mu_{B_j} \equiv \mu_{B_k} \equiv \mu_{B} = 11.8 \cdot 10^3 \text{ [bit / frame]} \tag{11}
\]

\[
\sigma^2_{B_i} \equiv \sigma^2_{B_j} \equiv \sigma^2_{B_k} \equiv \sigma^2_{B} = 60.87 \cdot 10^6 \text{ [bit}^2 / \text{frame}^2]\tag{12}
\]

that is, all the P- frames in the GoP have approximately the same statistical properties, as do the B- frames, independently of their position within the GoP. For this reason, in the rest of the paper we will refer without distinction to the P- and B- frames, not specifying their position.

In Fig. 2 the pdfs are shown for the whole sequence and for each subsequence respectively. Figs. 2a and 2b also show a visual comparison between the movie- and the I- pdfs with the Gamma functions with the same mean and variance. We can note that, as demonstrated in [20] through the Q-Q plot technique, the Gamma functions constitute a good approximation.

Fig. 3 shows the normalized autocorrelation functions of the movie trace and of the three subtraces. The movie-trace autocorrelation is plotted in a little interval in order to highlight some its properties. On account of the relationship between the mean frame sizes of the different frame types, the higher positive peaks correspond to the I- frames, the smaller positive ones to the P-frames, and the negative ones to the B- frames. A large I frame is followed by two small B-frames. Then a midsize P- frame is produced by the encoder, which is followed by two small B-
frames again. The pattern between two I-frame peaks is repeated with a slowly decaying peak amplitude.

As observed in [7][10][20], the MPEG video sequence presents self-similar behavior, testified by a Hurst parameter \( H = 0.884 \), calculated as average of the values obtained through the two methods obtained in Subsection 2.2. The I-subtrace is self-similar too, with a Hurst parameter \( H_I = 0.873 \) and also has an exponential decreasing short-range dependence, as we can see in Fig. 3b, comparing the ACf with the exponential function \( e^{\lambda_I h} \), \( \lambda_I = -0.891 \), which fits well up to a lag \( K_I = 30 \) frames. Therefore, the ACf of the I-subtrace is represented by the Hurst parameter \( H_I \) (LRD part), the exponent of the SRD part, \( \lambda_I = 0.0774 \) frame\(^{-1} \), and the knee, \( K_I \).

In order to demonstrate the need to take into account the intra-GoP correlation (i.e. the correlation between B-frames and the I-frame belonging to the same GoP, and between P-frames and the I-frame belonging to the same GoP) in achieving an MPEG model, Fig. 4 compares, for different values of utilization coefficient, the loss probability obtained by simulation in a buffer loaded by an MPEG-like trace taking into account the intra-GoP correlation and another trace constituted by the same I-frame subtrace and P- and B-frame subtraces generated with the same pdf, but not taking into account intra-GoP correlation. To highlight this important characteristic, we will investigate the distribution of the size of B- and P-frames once the I-frame size has been fixed. Let us define:

\[
a^{(B)}(r|r_I) = \lim_{h \to \infty} \text{Prob}\{B_k^{(i)} = r, k \in \{1, 2, 3, 4, 5, 6, 7, 8\} \mid I^{(i)} = r_I\} \quad (13)
\]

and

\[
a^{(P)}(r|r_I) = \lim_{h \to \infty} \text{Prob}\{P_k^{(i)} = r, k \in \{1, 2, 3\} \mid I^{(i)} = r_I\} \quad (14)
\]

The definition of these distributions, which refer to a frame type and not to \( k \), is correct thanks to the previous observation based on (9)-(12).

The above distributions are measurable for every fixed \( r_I \) but they are affected by measuring noise due to the small number of samples used. As an example, \( a^{(B)}(r|r_I) \) is plotted in Fig. 5 for \( r_I = 60,000 \). Nevertheless, in order to catch their nature, let us consider the distributions:

\[
a_k^{(B)}(r|r_I) = \lim_{h \to \infty} \text{Prob}\{B_k^{(i)} = r, k \in \{1, 2, 3, 4, 5, 6, 7, 8\} \mid I^{(i)} \in U_{\delta}(r_I)\} \quad (15)
\]

and
\[ a_{\delta}^{(P)}(r|r_I) = \lim_{h \to \infty} \text{Prob}\{P^{(k)}_i = r, \ k \in \{1, 2, 3\} \mid I^{(i)} \in U_\delta(r_I)\} \]  

(16)

where \( U_\delta(r_I) \) is the circular interval with center \( r_I \) and radius of \( \delta \), that is for \( \delta > 0 \), we have:

\[ U_\delta(r_I) = \{r : r \in [r_I - \delta, r_I + \delta]\} \]  

(17)

Observing Fig. 6, where \( a_{\delta}^{(B)}(r|r_I) \) is plotted for \( r_I = 60,000 \) and for different values of \( \delta \), we can see that, when \( \delta \) is not too small these distributions are well approximated by Gamma distributions, thus, since the following equations hold:

\[ a_{\delta}^{(B)}(r|r_I) = \lim_{\delta \to 0} a_{\delta}^{(B)}(r|r_I) \]  

(18)

\[ a_{\delta}^{(P)}(r|r_I) = \lim_{\delta \to 0} a_{\delta}^{(P)}(r|r_I) \]  

(19)

we can deduce that they have a Gamma nature.

Therefore, since a Gamma distribution is completely characterized by its mean and its variance, we need to study the mean value and the variance of the \( B \)- and \( P \)-frames once the \( I \)-frame is given, \( E\{B|I\} \) and \( \sigma_{B|I}^2 \), and \( E\{P|I\} \) and \( \sigma_{P|I}^2 \), respectively. These functions are plotted in Fig. 7. As we can see in this figure, the mean values of \( B \)- and \( P \)-frames and their variances are strictly dependent on the value of the \( I \)-frame belonging to the same GoP by a law which is very close to linear, so we can model these relations by means of a straight line whose equations:

\[ E\{B/I\} = 6033 \cdot I - 224.55 \]  

(20)

\[ E\{P/I\} = 125.928 \cdot I - 877.3 \]  

(21)

\[ \sigma_{B/I}^2 = 1.878 \cdot 10^6 \cdot I - 1.94 \cdot 10^8 \]  

(22)

\[ \sigma_{P/I}^2 = 7.475 \cdot 10^6 \cdot I - 6.48 \cdot 10^8 \]  

(23)

The coefficients of the previous straight line completely characterize the functions \( a_{\delta}^{(B)}(r|r_I) \) and \( a_{\delta}^{(P)}(r|r_I) \). The increasing trend of the above relations is due to the fact that large \( P \)-frames and large \( B \)-frames correspond to large \( I \)-frames and vice-versa. In fact, for example for the \( P \)-frames, if an \( I \)-frame is small in size, it means that, since a JPEG coding technique is applied to it, it is characterized by a high spatial redundancy; in this case, if no scene changes occur between this frame and the \( P \)-frames of the same GoP, these \( P \)-frames have also a high redundancy, and so for the difference between the \( I \)-frame and the \( P \)-frames of the same GoP. Therefore, since the information content transmitted in the \( P \)-frames is just this difference, the \( P \)-frames are small in
size also. Same reasoning can be done for the \( B \)-frames.

Let us now summarize our main findings and introduce an appropriate notation. The movie trace is constituted by a sequence of GoPs; let the \( i^{th} \) GoP, if constituted by 12 frames:

\[
I^i B_i(i) B_2(i) P_i(i) B_3(i) B_4(i) P_2(i) B_5(i) P_3(i) B_7(i) P_4(i) B_8(i)
\]

(24)

We have noticed that the statistical properties of the P- and B- frames are independent of their position within the GoP. The I- subtrace has a Gamma distribution and let \( f_I(x) \) be its pdf:

\[
f_I(x) = \frac{x^{m_I-1} e^{-x/\ell_I}}{\Gamma(m_I) \ell_I^{m_I}} \quad 0 < x < \infty \quad m_I, \ell_I > 0
\]

(25)

where \( m_I \) is the so-called “shape parameter” and \( \ell_I \) the “scale parameter”; they are related with the mean \( \mu_I \) and the variance \( \sigma^2_I \) of the I- subtrace as follows:

\[
m_I = \mu_I^2 / \sigma^2_I \quad \text{and} \quad \ell_I = \sigma^2_I / \mu_I
\]

(26)

The I- subtrace is self-similar; it is characterized by an SRD parameter \( \lambda_I \), an LRD parameter \( H_I \), and a “knee” \( K_I \). Therefore the autocovariance function, calculable from the autocorrelation function as \( C_{X_i,X_j}(m) = R_{X_i,X_j}(m) - \mu^2_i \), is of the following kind:

\[
C_{X_i,X_j}(m) = \begin{cases} 
  \exp(-\lambda_I m) & m \leq K_I \\
  L \cdot m^{-\beta_I} & m > K_I
\end{cases}
\]

(27)

Finally, we have observed that the mean values of \( B \)- and \( P \)-frames and their variances are strictly dependent on the value of the I-frame belonging to the same GoP by a law which is very close to linear, so we can model these relations by means of straight lines whose equations were given previously.

4. The algorithm

On the basis of what was said in Section 3, in an MPEG sequence, among the frames belonging to the same GoP, there is a correlation to be taken into account in defining a model for such traffic sources. In this section, our goal is to generate a trace with a pdf and ACf that closely match the corresponding functions of empirical traces and also to match the statistics for each kind of frame and the correlation relationships among them. The proposed algorithm is similar to those introduced in [11] and [13], but enhances them taking into account the intra-GoP correlation.
The inputs to the algorithm are, for the I-frames, as far as their ACF $R_{II}(h)$ is concerned, its knee, $K_I$, the Hurst parameter, $H_I$ (LRD behavior), and the exponent of the SRD part, $\lambda_I$, while, as far as the pdf $f_I(x)$ is concerned, the mean, $\mu_I$, and the variance, $\sigma^2_I$, enough to characterize a Gamma distribution. For the P- and B-frames, the inputs are the mean values, $\mu_p$ and $\mu_b$, the variances, $\sigma^2_p$ and $\sigma^2_b$, the cross-correlation coefficient between the I-frame and the P-frame processes, $\rho_{ip}$, and the cross-correlation coefficient between the I-frame and the B-frame processes, $\rho_{ib}$. Also, the desired number (even) of GoPs to be generated, $N$, is needed.

The algorithm steps are the following:

1. Obtain the ACF $\{\hat{R}_{II}(h)\}$, $h \in [1, N-1]$, from the ACF $R_{II}(h)$ as follows:
   $$\hat{R}_{II}(h) = R_{II}\left(\frac{N}{2} - h\right) \quad h \in [1, N-1]$$  \hspace{1cm} (28)
   symmetric about $N/2$.

2. Obtain a sequence $\{\chi_k^{(I)}\}$, $k \in [1, N-1]$, power spectrum estimate (or periodogram) of the I-subtrace, as the Fast Fourier Transform (FFT) of the autocorrelation sequence $\hat{R}_{II}(h)$. Let us recall that the periodogram $\{\chi_k^{(I)}(\omega)\}$ of a sequence $\{\chi_k\}$, $k \in [1, N-1]$, is linked to the Fourier Transform of this sequence, $Z^{(I)}(e^{j\omega})$, through the equation [21]:
   $$\chi_{N/2}^{(I)}(\omega) = \frac{1}{N/2} \left| Z^{(I)}(e^{j\omega}) \right|^2$$  \hspace{1cm} (29)

3. Construct $\{Z_k^{(I)}\}$, $k \in [1, N/2]$, a sequence of complex values such that $|Z_k^{(I)}| = \sqrt{\chi_k^{(I)} \cdot N/2}$, $k \in [1, N/2]$, and the phase of $Z_k^{(I)}$ uniformly distributed between 0 and $2\pi$. The random phase technique [22] preserves the power spectrum (and thus the autocorrelation) and ensures that different sample paths generated using the method will be independent.

4. Construct $\{\hat{z}_k^{(I)}\}$, $k \in [0, N-1]$, an “expanded” version of $\{Z_k^{(I)}\}$:
   $$\hat{z}_k^{(I)} = \begin{cases} 0 & \text{if } k = 0 \\ Z_k^{(I)} & \text{if } 0 < k \leq N/2 \\ \text{conjugate}(Z_{N/2+k}^{(I)}) & \text{if } N/2 < k < N \end{cases}$$  \hspace{1cm} (30)
The sequence \( \{z_k^{(r)}\} \) retains the power spectrum used in constructing \( \{Z_k^{(r)}\} \), but because it is symmetric about \( N/2 \), it now corresponds to the Fourier transform of a real-valued signal.

5. Use the inverse-Fourier transform \( \{z_k^{(r)}\} \) to obtain the synthesized trace \( \{I_k^{(r)}\} \), \( k \in [0, N-1] \), with the required autocorrelation function.

6. Impose the pdf \( f_I(x) \) on the sequence \( \{I_k^{(r)}\} \); let \( f_{I_k^{(r)}} \) be the pdf of the process obtained in step 5. We can generate the process \( \{I_k^\prime\} \) with the desired pdf by using the following transformation [23]:

\[
I_k^\prime = h(I_k^{(r)}) = F_{i_k^{(r)}}^{-1}(F_{i_k}(I_k^{(r)})) \quad k = 1, 2, \ldots
\] (31)

where \( F_{i_k^{(r)}} \) and \( F_{i_k} \) are the marginal cumulative probability functions of the empirical process \( \{I\} \) to be synthesized and of the process \( \{I_k^{(r)}\} \) obtained in step 4. In [24] it has been demonstrated that the transformation in (31) preserves the Hurst parameter, that is, the heavy-tail behavior of the process \( \{I_k^\prime\} \) is the same as that of the process \( \{I_k^{(r)}\} \) and, therefore, of the target process \( \{I_k\} \).

7. Impose again the autocorrelation function altered in the previous step. If we indicate the autocorrelation functions of the processes \( \{I_k^\prime\} \) and \( \{I_k^{(r)}\} \) as \( R_{\gamma_i(h)}(h) \) and \( R_{\gamma_i}(h) \), it can be shown [11] that

\[
\frac{R_{\gamma_i(h)}}{R_{\gamma_i}(h)} = a \quad \text{as} \ h \to \infty
\] (32)

where

\[
a = \frac{E\{I_k^\prime I_k^\prime\}}{E\{(I_k^{(r)})^2\}}
\] (33)

is the “attenuation factor” and \( 0 < a \leq 1 \). Once the parameter \( a \) has been obtained from (33), we construct a process \( \{I_k^{\prime\prime}\} \) with an autocorrelation function \( R_{\gamma_i}(h) \) coinciding with \( R_{\gamma_i}(h) \) in the short range, and with \( R_{\gamma_i}(h)/a \) in the long range. As far as the short term part is concerned, since the behavior of \( R_{\gamma_i}(h) \) is approximately exponentially decreasing of the kind \( R_{\gamma_i}(h) = e^{-\gamma_i h} \), for \( h < K_{\gamma_i} \), to get the parameter \( \gamma_i \) we solve the following equation, establishing that the function \( R_{\gamma_i}(h) \) is a continuous one in \( h = K_{\gamma_i} \):
\[ e^{-\tau_i K_i} = R_H(K_i) / a \]  \hfill (34)

8. Re-apply steps 1 to 6 to the process \( \{I''_k\} \), to find the process \( \{I_k\} \), \( k \in [1, \ldots, N] \), with the required pdf and ACf. This process represents the synthesized sequence of the I-frames.

9. Generate, for the generic \( k^{th} \) GoP, \( k \in [1, \ldots, N] \), the other three P-frames and the eight B-frames, taking into account that the I-frame of this GoP is \( I_k \), generated in the previous steps. Obviously, the whole generated sequence for the MPEG movie has \( 12 \cdot N \) elements.

5. **Model evaluation**

The algorithm outlined in the previous section can be applied to generate traces with the same intra- and inter-GoP characteristics of any MPEG movie. In this section, as an example, we apply this algorithm to generate a trace synthesizing the MPEG video sequence of the movie “The Simpsons”, analyzed in Subsection 3.2.

The comparison will be made for each type of frame, that is, on the single subtraces, and for the whole sequence. Since the algorithm is more efficient (in the FFT operations) if the number of I-frames is a power of 2, we will generate \( N = 16384 \) I-frames.

Fig. 8 shows the symmetric ACf \( \tilde{R}_H(h) \) obtained in step 1 of the algorithm. In Fig. 9, showing the ACf of the trace obtained through the steps 1-5, \( R_{I''}(h) \), compared with the required ACf \( R_I(h) \), we can appreciate the goodness of the proposed fast algorithm to generate a trace with a given ACf.

In Fig. 10 the pdf of the sequence \( \{I_k'\} \) (before step 6) and the pdf of the sequence \( \{I''_k\} \) (after step 6) are compared with the required pdf \( f_I(x) \).

In step 7 we obtain the attenuation factor \( a = 0.9738 \) and the parameter \( \gamma = 0.739 \), to be next used in step 8 to obtain the definitive I-frame trace \( \{I_k\} \), whose pdf and ACf are plotted in Fig. 11, compared with the required ones. Finally, through step 9, we obtain the whole trace. This trace has a Hurst parameter \( H = 0.88 \), the same mean and variances taken as inputs. The comparison between the pdf of the obtained trace and the pdf \( f_I(x) \), plotted in Fig. 2a, has been done with a Q-Q plot which compares the quantiles of two data sets. If the two data sets are identical, the quantiles will fall on the \( 45^\circ \) line; if the data sets are close, the quantiles will be near this line. The obtained Q-Q plot is shown in Fig. 12. Analyzing the traces for each kind of frame, we measure an I-frame \( H \) parameter \( H_I = 0.869 \) and, as was expected, they get the same values.
of the means, variances, and coefficients of the straight-lines listed in (20)-(23), taken as inputs for the algorithm.

6. Conclusions

In this paper after demonstrating that intra-GOP correlation, that is, the correlation between frames belonging to the same GoP, has to be taken into account, it is proposed an accurate model allowing the generation of an MPEG-like trace. Moreover, a new fast algorithm for the generation of self-similar traces has been proposed. This algorithm allows to generate a trace with the same SRD and LRD behavior and the same probability density function of any video sequence. As an example, the proposed algorithm has been used to generate a trace with the same statistical characteristics of the movie “The Simpsons” and the results have been good, also in the CPU time needed in the computation point of view.
References


### Table 1: Parameters measured from the “The Simpsons” sequence: movie trace

<table>
<thead>
<tr>
<th></th>
<th>Movie trace</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$ [bit / frame]</td>
<td>81.9 · 10^3</td>
</tr>
<tr>
<td>$\sigma^2$ [bit^2 / frame^2]</td>
<td>520 · 10^6</td>
</tr>
<tr>
<td>$H$</td>
<td>0.884</td>
</tr>
</tbody>
</table>

### Table 2: Parameters measured from the “The Simpsons” sequence: I subtrace

<table>
<thead>
<tr>
<th></th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$ [bit / frame]</td>
<td>81.9 · 10^3</td>
</tr>
<tr>
<td>$\sigma^2$ [bit^2 / frame^2]</td>
<td>453 · 10^6</td>
</tr>
<tr>
<td>$H$</td>
<td>0.873</td>
</tr>
<tr>
<td>SRD coefficient: $\lambda$</td>
<td>-0.891</td>
</tr>
<tr>
<td>Knee: $K$ [frames]</td>
<td>30</td>
</tr>
</tbody>
</table>

### Table 3: Parameters measured from the “The Simpsons” sequence: P subtraces

<table>
<thead>
<tr>
<th></th>
<th>P1 trace</th>
<th>P2 trace</th>
<th>P3 trace</th>
<th>average P</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$ [bit / frame]</td>
<td>23.98 · 10^3</td>
<td>23.97 · 10^3</td>
<td>23.96 · 10^3</td>
<td>23.97 · 10^3</td>
</tr>
<tr>
<td>$\sigma^2$ [bit^2 / frame^2]</td>
<td>297.44 · 10^3</td>
<td>292.93 · 10^3</td>
<td>302.67 · 10^3</td>
<td>297.68 · 10^3</td>
</tr>
</tbody>
</table>

### Table 4: Parameters measured from the “The Simpsons” sequence: B subtraces

<table>
<thead>
<tr>
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<th>B3</th>
<th>B4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$ [bit / frame]</td>
<td>11.92 · 10^3</td>
<td>11.81 · 10^3</td>
<td>11.81 · 10^3</td>
<td>11.89 · 10^3</td>
</tr>
<tr>
<td>$\sigma^2$ [bit^2 / frame^2]</td>
<td>58.82 · 10^3</td>
<td>55.91 · 10^3</td>
<td>59.63 · 10^3</td>
<td>77.75 · 10^3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>B5</th>
<th>B6</th>
<th>B7</th>
<th>B8</th>
<th>average B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$ [bit / frame]</td>
<td>11.81 · 10^3</td>
<td>11.71 · 10^3</td>
<td>11.95 · 10^3</td>
<td>11.79 · 10^3</td>
<td>11.83 · 10^3</td>
</tr>
<tr>
<td>$\sigma^2$ [bit^2 / frame^2]</td>
<td>59.45 · 10^3</td>
<td>56.63 · 10^3</td>
<td>62.33 · 10^3</td>
<td>56.47 · 10^3</td>
<td>60.87 · 10^3</td>
</tr>
</tbody>
</table>
Figure 1: Group of Pictures (GoP) of an MPEG trace

- a): Movie trace
- b): I- trace
- c): P- trace
- d): B- trace

compared with the gamma functions with the same means and variances

Figure 2: Probability density function
a): Movie trace for lag $h \in [1,96]$  

b): I-trace and short range fitting

c): P- trace  

d): B-trace

**Figure 3:** Autocorrelation function
(a) utilization coefficient $\rho = 0.4$

(b) utilization coefficient $\rho = 0.5$

(c) utilization coefficient $\rho = 0.6$

(A): With intra-GoP correlation  (B): Without intra-GoP

**Figure 4**: Loss probability comparison

**Figure 5**: An example of the $a^{(3)}(r|\rho)$ function when $r = 60,000$
Figure 6: Three examples of $d_{\delta}^{(B)}(r|I)$ for $r_I = 60,000$ and $\delta = 1000$ (a), $\delta = 100$ (b) and $\delta = 10$ (c).

Figure 7: Statistics of $d^{(B)}(r|I)$ and $d^{(P)}(r|I)$ compared with the best-fitting straight lines.

Figure 8: Autocorrelation function $\hat{R}_{\beta}(h)$ obtained in step 1.

Figure 9: Comparison of $R_{1/\nu}(h)$ with the required ACF $R_{\beta}(h)$.
Figure 10: Comparison of $f_{x'}(x)$ (before step 6) and $f_{x''}(x)$ (after step 6) with the required pdf $f_{x}(x)$

Figure 11: Comparison of the pdf and ACf of the generated I-trace with the required $f_{y}(h)$ and $R_{xy}(h)$
Figure 12: Q-Q plot of the pdf of the generated whole trace and the movie-trace pdf $f(x)$