June 27th 2011
Exam of Switching architectures

**Rules for the exam.** It is forbidden to use notes, books or calculators. When needed, use approximations. Report the solution of each problem on a different paper sheet.

**Time available:** 70 minutes.

**PROBLEM A**
Design a rearrangeable switch of size $800 \times 350$ using only modules of size $10 \times 10$, with the aim of minimizing the number of modules.

1. Describe the architecture
2. Compute the total number of modules required
3. Describe the configuration algorithm
4. Write the formula to compute the minimum theoretical number of modules to build the switch.

**PROBLEM B**
Consider a $3 \times 3$, bufferless switch, fed by non-uniform Bernoulli i.i.d. arrivals according to the following rate matrix:

\[
\Lambda = \rho \begin{pmatrix}
\frac{1}{3} & \frac{1}{6} & \frac{1}{3} \\
\frac{1}{3} & \frac{1}{3} & 0 \\
\frac{1}{2} & 0 & 0
\end{pmatrix}
\]

where $\rho$ is the normalized load at each input. Assume that contentions among packets directed to the same output are solved at random. Compute analytically

1. the traffic admissibility conditions;
2. the throughput measured for each output port in function of $\rho$;
3. the maximum throughput achievable under admissible traffic;
4. the corresponding loss probability for a packet destined to output 1;
5. the corresponding loss probability for a packet destined to output 3.

**PROBLEM C**
Prove that the weight of a greedy maximum weight matching (GWM) is at least equal to half the weight of the maximum weight matching (MWM). In other words, $W(GWM) \geq \frac{1}{2} W(MWM)$.

In the proof, denote by $E$ the set of edges in the bipartite graph, by $G$ the sub-set of edges selected by the GWM, and by $M$ the sub-set of edges selected by the MWM scheduler.