July 19th 2010

Exam of Switching architectures

Rules for the exam. It is forbidden to use notes, books or calculators. When needed, use approximations. Report the solution of each problem on a different paper sheet.

Time available: 70 minutes.

PROBLEM A
Consider a slotted input queued switch, of size $N \times M$ and with Virtual Output Queueing. The switch is fed by variable-size packets.

1. Describe in pseudo-code a scheduling algorithm working in cell-mode.
2. Describe in pseudo-code a scheduling algorithm working in packet-mode.
3. Compare the performance of the cell-mode algorithm and the packet-mode algorithm in terms of throughput and delay.

PROBLEM B
Consider a $4 \times 4$, bufferless switch, fed by non-uniform Bernoulli i.i.d. arrivals according to the following rate matrix:

$$
\Lambda = \rho \begin{pmatrix}
\frac{1}{3} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} \\
\frac{1}{3} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} \\
\frac{1}{3} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} \\
0 & 0 & 0 & 0
\end{pmatrix}
$$

where $\rho$ is the normalized load at each input. Assume that contentions among packets directed to the same output are solved at random. Compute analytically

1. the traffic admissibility conditions;
2. the throughput measured for each output port in function of $\rho$;
3. the maximum throughput achievable under admissible traffic;
4. the loss probability for a packet destined to output 1;
5. the loss probability for a packet destined to output 3.

PROBLEM C
Design a $8 \times 8$ Cantor network.

1. What are the properties of such network?
2. Draw the complete network.
3. Describe the algorithm to configure the network.
4. Configure the network following exactly the following sequence of input-output couples: $4 \rightarrow 1, 5 \rightarrow 6, 6 \rightarrow 2, 7 \rightarrow 4, 8 \rightarrow 3, 1 \rightarrow 5, 2 \rightarrow 8, 3 \rightarrow 7$. 
SOLUTIONS

Problem A

// CELL-MODE SCHEDULER
// Q[i][j] is the number of cells in VOQ[i][j]
int matching[N] // m[i]=j if input i is connected to output j; else NOT_USED
int output_reserved[M] // TRUE/FALSE

// init
for (j=0; j<M; j++)
    output_reserved[j]=FALSE
for (i=0; i<N; i++)
    matching[i]=NOT_USED

// schedule
for (i=0; i<N; i++) // for each input
    for (j=0; j<M; j++) // for each output
        if (Q[i][j]>0 && output_reserved[j]==FALSE)
            // found available HoL HEAD cell
            matching[i]=j
            output_reserved[j]=TRUE
            break

// PACKET-MODE SCHEDULER
// Q[i][j] is the number of cells in VOQ[i][j]
// S[i][j] is the state of head-of-line cell of VOQ[i][j]: HEAD/MIDDLE/TAIL
static int matching[N] // m[i]=j if input i is connected to output j; else NOT_USED
int output_reserved[M] // TRUE/FALSE

// init
for (j=0; j<M; j++)
    output_reserved[j]=FALSE

// check old matching
for (i=0; i<N; i++)
    j=matching[i]
    if (j>=0 && Q[i][j]>0 && S[i][j]!=HEAD)
        output_reserved[j]=TRUE // keep old edge
    else
        matching[i]=NOT_USED // for empty queues or HEAD-cells

// schedule
for (i=0; i<N; i++) // for each input
    if (matching[i]==NOT_USED) // check if the input is still available
        for (j=0; j<M; j++) // for each output
            if (Q[i][j]>0 && output_reserved[j]==FALSE)
                // found available HoL HEAD cell
                matching[i]=j
                output_reserved[j]=TRUE
                break

Problem B

The admissibility conditions are $\rho \leq 1$.
Consider a generic timeslot. Let $X_1$ be the number of cells arrived and directed to output 1 and 2 and let $X_2$ be the number of cells arrived and directed to output 3 and 4.

$$P(X_1 = 0) = \left(1 - \frac{\rho}{3}\right)^3 \quad P(X_2 = 0) = \left(1 - \frac{\rho}{6}\right)^3$$

Now the throughput for outputs 1 and 2 is:

$$T_1(\rho) = P(X_1 \geq 1) = 1 - P(X_1 = 0) = 1 - \left(1 - \frac{\rho}{3}\right)^3$$

and for outputs 3 and 4:

$$T_2(\rho) = P(X_2 \geq 1) = 1 - P(X_2 = 0) = 1 - \left(1 - \frac{\rho}{6}\right)^3$$
The maximum throughput is achieved for $\rho = 1$:

$$T_1(1) = 1 - (1 - 1/3)^3 \approx 0.70 \quad T_2(1) = 1 - (1 - 1/6)^3 \approx 0.42$$

Consider now a packet $p$ arrived at input 1 and destined to output 1. Let us compare $p$ with the packet at input 2. The probability that input 2 has a packet for the same output is $\rho/3$. In such case, the probability that $p$ is lost is $1/2$, and hence the final probability that $p$ is not lost is $1 - \rho/6$. After this, compare $p$ with the packet at input 3. Also in this case, the final probability that $p$ is not lost is $1 - \rho/6$. As a consequence, the final probability that $p$ is not lost is $(1 - \rho/6)^2$ and the loss probability is $1 - (1 - \rho/6)^2$.

Analogously, the loss probability for a packet destined to output 3 is $1 - (1 - \rho/12)^2$. 