Rules for the exam. It is forbidden to use notes, books or calculators. When needed, use approximations. Report the solution of each problem on a different sheet of paper.

Time available: 70 minutes.

PROBLEM A
Consider a $4 \times 4$ input queued switch, with each port running at 8 Gbps. Assume that the internal timeslot corresponds to a 64 bytes packet. The following rate matrix must be guaranteed:

$$\hat{R} = \begin{bmatrix} 0 & 1 & 4 & 2 \\ 1 & 2 & 0 & 1 \\ 4 & 0 & 1 & 1 \\ 2 & 2 & 2 & 2 \end{bmatrix} \text{ Gbps}$$

1. Use the Paul algorithm to find the possible frame sequence, named $F_1$.
2. Use the Birchoff von Neumann decomposition to find the possible frame sequence, named $F_2$.
3. Are $F_1$ and $F_2$ the same? Why?
4. Under which admissibility conditions, the two frame sequences $F_1$ and $F_2$ allow to obtain the maximum throughput?
5. Rearrange $F_1$ and $F_2$ to maximize the worst case access delay, under low traffic load, for the flow from input 1 to output 3. Compute this delay in $\mu$s.
6. Rearrange $F_1$ and $F_2$ to minimize the worst case access delay, under low traffic load, for the flow from input 1 to output 3. Compute this delay in $\mu$s.

PROBLEM B
Consider two scheduling algorithms $S_1$ and $S_2$, for an $N \times N$ input queued switch. Let $S_1$ compute a maximal weight matching. Let $S_2$ exploit memory from past matchings.

1. Describe in pseudo-code the algorithms $S_1$ and $S_2$, assuming that it is already available a function, returning a random matching, declared as follows:

   ```c
   int *create_random_matching(void)
   ```

2. Describe the sufficient conditions for $S_1$ and $S_2$ to obtain 100% throughput.
3. Compute the approximated computational complexity for $S_1$ and $S_2$ in terms of elementary operations, knowing that the minimum complexity to find a random matching is $O(N \log N)$.

PROBLEM C
In order to discuss the complexity of building an asymmetric $N \times \alpha N$ switch, with $\alpha \in (0, 1)$, answer to the following questions:

1. Compute the number of switching configurations supported by the switch.
2. What is the complexity reduction with respect to an $N \times N$ switch when adopting an optimal theoretical architecture? What is the complexity reduction obtained by adopting the crossbar architecture?

If needed, use the Stirling approximation:

$$N! \sim \sqrt{2\pi N} \left(\frac{N}{e}\right)^N$$