Capacity Scaling in Ad Hoc Networks with Heterogeneous Mobile Nodes: the Sub-critical Regime

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***Abstract*— In this paper we provide the scaling laws for the transport capacity of a wide class of mobile wireless ad hoc networks. Our analysis generalizes previous results obtained under restrictive assumptions on the node mobility process and overall node density over the network area. The broader family of mobile networks that we consider is able to account for many important characteristics usually recognized in real traces of both human and vehicular mobility. In particular, we consider clustered networks of heterogeneous nodes, in which the shape of the spatial distribution of each node around one or more home-points plays a fundamental role in determining the overall transport capacity. We identify different operational regimes that arise within our general class of mobile networks, and for each regime we characterize the asymptotic network capacity.**

***Index Terms*— Mobile Ad Hoc Networks, Performance Analysis, Scaling Laws.**

I. INTRODUCTION

STORE-and-forward has been for decades the fundamental communication paradigm of packet switched networks, including the first generation of ad hoc wireless networks originally conceived at the end of the last century [1]. The routing protocols developed for these networks have been traditionally based on the assumption that the network topology is always connected, so that, at any time, there exists a path from any sender to its intended receiver. In this context, topological changes due to node mobility have been invariably seen as something evil or at least undesirable, which inevitably degrades the performance achieved in an otherwise stable network topology. In recent years, this traditional view of packet switched networks has changed, as node mobility has become a necessary, fundamental component of a new kind of networks based on a novel store-carry-and-forward communication paradigm (also named "encounter-based forwarding" or "mobility-assisted routing"). According to this new paradigm, relay nodes can physically carry buffered data while they move around the network area, till they get in contact with a suitable next-hop node. Although this scheme incurs much longer delays than the traditional store-and-forward scheme, on the time scale of node movement across the network space, it has laid the foundation of an entire new area of research, usually referred to as Delay Tolerant Networking (DTN) [2], [3], which has recently attracted a lot of attention. Interesting applications include "pocket switched networks" based on human mobility [4], vehicular networks based on public buses [5] or taxicabs [6], sensor networks for wildlife tracking [7], rural kiosks providing Internet access in developing nations [8].

The reason why node mobility plays a fundamental role in DTN is two-fold. Firstly, it permits end-to-end communication in the first place, in the case of sparse, intermittently connected networks suffering from frequent partitioning of the nodes. Indeed, although instantaneous end-to-end routes do not always exist, messages can nevertheless be delivered over time, through the sequence of connectivity graphs generated by nodes' movement. Secondly, mobility can dramatically increase the overall transport capacity of the network, allowing it to scale up to very large number of nodes. Indeed, interference-limited networks of static nodes are known to suffer severe per-node throughput decay (in the order of $1/\sqrt{n}$) as the number of nodes n goes to infinity [9]. In contrast, a simple two-hop relay scheme can keep the per-node throughput constant, under an ideal mobility process in which each node independently and uniformly visits the entire network space [10].

Several routing protocols for sparse delay tolerant networks have been proposed in the literature, which face the challenging problem of selecting good routes in space and time using very limited (and rapidly changing) information about the system state [11]–[16]. However, only few studies have analyzed the fundamental scaling laws of the maximum achievable network capacity in the presence of heterogeneous nodes and under different mobility models. Indeed, the static scenario analyzed by Gupta-Kumar [9] and the 'homogeneous mixing' scenario considered by Grossglauser-Tse [10] are two particular species of a rich zoo of wireless ad hoc networks exhibiting a variety of scaling behavior as we change the mobility patterns of the nodes. The goal of this paper is to provide a comprehensive overview of capacity results for a wide class of mobile networks which accounts for many characteristics largely recognized in real mobility traces of people, animals, and vehicles.

The consideration of the fact that an individual node may not move over the entire network area, but just in a small portion of it, has already pushed some researchers to analyze the impact of restricted mobility models. In [17] the authors extend their results in [10] considering the case where each node independently moves along a randomly chosen great circle on the sphere of unit surface. Quite surprisingly, even under this one-dimensional mobility pattern a constant throughput per source-destination pair can be sustained as the number of nodes increases. In [18] the network of unit area is partitioned into square cells, and nodes are restricted to move within one randomly chosen cell; the authors consider two cases in which the cell area either scales as $(\log n)/n$ or remains constant, obtaining results similar to the scenarios of Gupta-Kumar and Grossglauser-Tse, respectively. The impact of inhomogeneous node density (e.g., node clusters) has been

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analyzed in [19], [20] for the case of static networks only. In [21], [22] we have addressed the general problem of determining the capacity of mobile networks in the presence of heterogeneous nodes and anisotropic mobility. The contributions of this paper with respect to our previous work will be explained in detail in Section III, after introducing the class of wireless networks under study.

II. System Assumptions and Notation

A. Mobility Model

1) **The basic scenario:** The family of mobile networks that we first analyze in Sec. IV is denoted by $\mathcal{F}(\alpha, \delta, \nu)$ and is characterized by three parameters α, δ, ν. For a given choice of parameters, we build a sequence of networks $\mathcal{F}_n$ indexed by the number of nodes n, and let n go to infinity. Nodes are supposed to move over a bidimensional Torus surface $\mathcal{O}_n$ of size L_n x L_n (to avoid border effects). As we add more and more nodes into the system, the network physical extension L_n is expected to increase. To account for this fact, we assume that L_n scales with n as n^α, with $0 \leq \alpha \leq 1/2$. The extreme cases are: $\alpha = 0$, for which the network area remains constant, and average node density increases linearly with n; $\alpha = 1/2$, for which the network area increases linearly with n, and average node density remains constant. The first parameter of our network family, α, thus determines how the network area (or, alternatively, the average node density) behaves as we increase the number of nodes.

Anisotropic mobility and node heterogeneity are taken into account in our model as follows. First, we observe that, as proven in [21], the transport capacity of a mobile network depends on the mobility pattern only through the stationary distribution of the nodes over the area, under the assumption that the mobility processes of the nodes are jointly stationary and ergodic. In this work, we further assume that nodes move independently of each other. Hence, we only need to characterize the shape of the spatial distribution of individual nodes over region $\mathcal{O}_n$. We consider that each node has one spot where it is most likely to be found and we refer to such spot as the "home-point" of the node. This can be the private home or the workplace in the case of people, the garage or the warehouse in the case of vehicles, the nest or lair in the case of animals. We assume that a node moves around its home-point according to a general ergodic process which results into a rotationally-invariant spatial distribution $\phi(d)$ which decays with the distance[1] d from the home-point as a power law of exponent δ, with $\delta \geq 0$.

To avoid convergence problems in proximity of the home-point, we take function $s(d) = \min(1, d^{-\delta})$, and normalize it so as to obtain a proper probability density function over the network area: $\phi(d) = \frac{s(d)}{\iint_{\mathcal{O}_n} s(d)}$. The precise value of the normalization constant $G(n) = \iint_{\mathcal{O}_n} s(d)$ is cumbersome to derive due to the square shape of the network area (as opposed to the circular symmetry of $\phi(d)$), however it can be approximated, in order sense [2], by the following integral in polar coordinates:

$$G(n) = \Theta\left(\int_0^{2\pi} \mathrm{d}\theta \int_0^{L_n/2} \min(1, \rho^{-\delta}) \rho \, \mathrm{d}\rho\right)$$

We obtain that $G(n)$ is finite for any $\delta > 2$. For $0 \leq \delta < 2$ we have $G(n) = \Theta(L_n^{2-\delta})$. Let $E[d]$ be the average distance reached by the node from the home-point. The value of $E[d]$ can be approximated (in order sense) by

$$E[d] = \Theta\left(\frac{1}{G(n)} \int_0^{2\pi} \mathrm{d}\theta \int_0^{L_n/2} \min(1, \rho^{-\delta}) \rho^2 \, \mathrm{d}\rho\right)$$

We have that $E[d]$ is finite for any $\delta > 3$. For $2 < \delta < 3$, it results[3] $E[d] = \Theta(L_n^{3-\delta})$, whereas for $0 \leq \delta < 2$ we have $E[d] = \Theta(L_n)$.

Parameter δ accounts for the fact that an individual node does not visit uniformly the network area, but spends most of the time just in a limited portion of it. However, parameter δ alone does not allow to control the overall node density over the area. In many realistic scenarios it has been found that the node density is largely inhomogeneous, and typically exhibits concentration points [6] or hotspots [23] where nodes are more likely to gather. Such clustering behavior has been observed in many different traces related to both human and vehicular movements, and appears to be a quite ubiquitous feature of realistic mobility processes. Concentration points are usually well distinct from each other and fairly stable over time. Examples include dormitories, conference rooms, restaurants, movie theaters, (in the case of people), intersections, parking lots, gas stations (in the case of vehicles), watering holes, oases (in the case of animals).

In our mobility model, we introduce clustering through the distribution of the nodes' home-points. In particular, we consider two different models:

- **Uniform**: home-points of nodes are uniformly and independently chosen inside area $\mathcal{O}_n$.
- **Clustered Random**: each home-point, independently of others, chooses one of m clusters, all clusters being equally likely. Each cluster has a middle point which is uniformly located within $\mathcal{O}_n$. The home-points of the same cluster are then uniformly and independently placed within a disk of radius r centered at the cluster middle point.

In the Clustered Random model, we let the number of clusters scale with n, defining $m = n^\nu$, with $0 < \nu < 1$. The Uniform model can be considered as an extreme case of the Clustered Random model when $\nu = 1$ and, in addition, each cluster contains deterministically one home-point. We introduce the cluster density over the area $\rho_C = n^\nu / n^{2\alpha} = n^{\nu - 2\alpha}$ and the average inter-distance between cluster middle points $d_C = n^{\alpha - \nu/2}$. Finally, we assume that the cluster radius $r = o(d_C)$, so that, with high probability, clusters do not overlap. Examples of home-point distributions according to Uniform and Clustered Random models are shown in Fig. 1 for $n = 10,000$.

In the following we will also consider the **Clustered Grid** model, which differs from the Clustered Random model in that: i) the cluster middle points are regularly placed over $\mathcal{O}_n$, according to a square grid with step d_C; ii) $r = 0$, i.e., all home-points are co-located at the cluster middle-point. The Clustered Grid model allows to simplify the presentation of some of the ideas of the paper, and constitutes an intermediate step towards the analysis of the Clustered Random model.

[1]Given any two points $X_1 = (x_1, y_1) \in \mathcal{O}_n$ and $X_2 = (x_2, y_2) \in \mathcal{O}_n$ we define their distance according to: $d(X_1, X_2) = \min_{u,v \in \{-L_n, 0, L_n\}} \sqrt{(x_1 + u - x_2)^2 + (y_1 + v - y_2)^2}$

[2]Given two functions $f(n) \geq 0$ and $g(n) \geq 0$: $f(n) = o(g(n))$ means $\lim_{n\to\infty} f(n)/g(n) = 0$; $f(n) = O(g(n))$ means $\limsup_{n\to\infty} f(n)/g(n) = c < \infty$; $f(n) = \omega(g(n))$ is equivalent to $g(n) = o(f(n))$; $f(n) = \Omega(g(n))$ is equivalent to $g(n) = O(f(n))$; $f(n) = \Theta(g(n))$ means $f(n) = O(g(n))$ and $g(n) = O(f(n))$; at last $f(n) \sim g(n)$ means $\lim_{n\to\infty} f(n)/g(n) = 1$

[3]In this paper we will not consider the two special cases $\delta = 2$ and $\delta = 3$

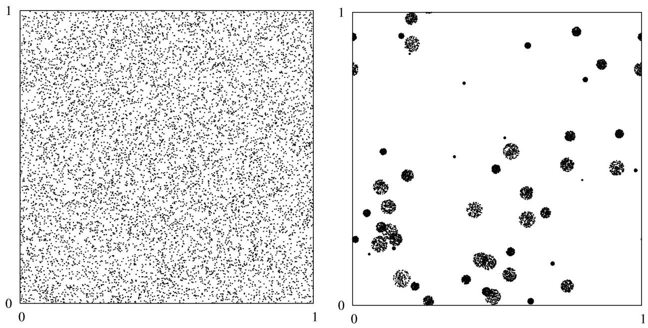

Fig. 1. Examples of home-point distributions according to Uniform model (left plot) and Clustered Random model (right plot), in the case of $n = 10,000$ nodes.

Discussion: The class of mobile networks that we study is very general and encompasses a wide range of possible scenarios, while accounting for what we think are ubiquitous features of realistic mobility processes (clustering and power laws in the spatial distribution of nodes). Notice that the Grossglauser-Tse scenario corresponds to the subclass $\mathcal{F}(-, 0, -)$ in which $\delta = 0$, whereas the other parameters can take any value. The case of static nodes uniformly deployed over the area (Gupta-Kumar scenario) can be represented by $\mathcal{F}(1/2, \infty, 1)$, i.e., a Uniform model ($\nu = 1$) in which the mobility of each node around its home-point is so limited ($\delta \to \infty$) that the node can be considered to be static, while the node density over the area is kept constant ($\alpha = 1/2$).

The use of power laws for the node spatial distribution is justified by a number of measurement studies: in [6], the authors analyze a large mobility trace of taxis in the city of Warsaw. The empirical distribution of the number of taxis falling in the cells of a regular grid is found to be heavy-tailed and fairly stable over time. In [24] the duration of contact times between people is found to be heavy-tailed in different traces related to conference and campus-wide experiments. In [25] authors analyze a corporate wireless local area network and find that the fraction of time spent by users with a given access point exhibits a power law.

2) **Extension to multiple home-points:** In Sec. V we generalize the basic scenario by allowing each node to have multiple home-points. At any given time we say that a node is associated to one of its home-point, meaning that it moves around it in the same way as in the basic case, i.e., according to a general ergodic process which results into a rotationally-invariant spatial distribution $\phi(d)$. We assume that each node periodically migrates from one of its home-point to another, and that the fraction of time during which the node is associated to one of its home-points is constant and non null. Moreover, we consider that the time taken to switch from one home-point to another is negligible, hence the node does not have the opportunity of exchanging data with other nodes while traveling from one home-point to another other (the contribution of such data exchanges to the asymptotic network capacity is, in any case, not important in order sense).

Multiple home-points are assigned to a given node i according to the following procedure. First, we randomly select one home-point in the area to be the *primary* home-point of the node, and denote it by X_i^h. Then we specify the probability distribution of the distance z between the primary home-point and any other of its home-points, assuming that the direction is uniformly distributed in $[0, 2\pi]$. Since the number of home-points of a node is finite, it turns out that all of the gain in terms of network capacity is achieved (in order sense) in the case of just two home-points. Hence, we limit ourselves to considering only one *secondary* home-point, located at Y_i^h. As an example, the primary home-point could be the residence of a person, and the secondary home-point his/her workplace. If the primary and secondary home-points of a node belong to the same cluster, or, more in general, they are separated by a distance $o(d_C)$, the behavior is the same as in the case of a single home-point, hence there is no gain in terms of network capacity. The only interesting case is when the distance between the two home-points of a node is $\Omega(d_C)$, therefore we assume that the distribution of the distance z between them is according to $\min(1, (z/d_C)^{-\zeta})/Z$, where Z is a proper normalization factor and $\zeta \geq 0$ is one additional parameter of our model. When $\zeta > 2$, the average distance $E[z]$ between the two home-points is $\Theta(d_C)$. For $1 < \zeta < 2$, $E[z]$ behaves as $\Theta(L_n/n^{\nu/2(\zeta-1)})$, whereas for $\zeta < 1$ we have $E[z] = \Theta(L_n)$. Therefore, varying ζ between 1 and 2, we relate $E[z]$ to the physical network extension, obtaining all intermediate cases between d_C (the minimum value for which multiple home-points are beneficial) and L_n (equivalent to choosing the secondary home-point uniformly in the area).

3) **Extension to anisotropic mobility patterns:** In Sec. VI we relax the assumption that the spatial distribution of a node is rotationally-invariant around the home-point, extending the analysis to the case of anisotropic mobility patterns. In particular, restricting ourselves to the single home-point case[4], we consider the situation in which the spatial distribution of a node is described by function

$$\phi(d_1, d_2) = \frac{\min(1, d_1^{-\delta_1})}{G_1(n)} \frac{\min(1, d_2^{-\delta_2})}{G_2(n)}$$

where d_1 and d_2 are orthogonal components of the distance vector from the home-point, while δ_1 and δ_2 are the power-law exponents of the marginal distribution along the corresponding direction. The normalization constant $G_1(n)$ (or $G_2(n)$) can be approximated (in order sense) by the uni-dimensional integral of $\min(1, d_1^{-\delta_1})$ over the interval $[0, L_n]$. We have that $G_1(n)$ is finite for any $\delta_1 > 1$. For $0 < \delta_1 < 1$ we have $G_1(n) = \Theta(L_n^{1-\delta_1})$ (similarly for $G_2(n)$). Let $E[d_i] = \int_{\mathcal{O}_n} d_i \phi(d_1, d_2)$ be the average distance reached by a node from its home-point along direction $i = 1, 2$; $E[d_i]$ is finite for any $\delta_i > 2$; $E[d_i] = \Theta(L_n^{2-\delta_i})$ for $1 < \delta_i < 2$, and, at last, $E[d_i] = \Theta(L_n)$ for $0 \leq \delta_i < 1$.

Given a pair of values δ_1 and δ_2, we identify two sub-cases depending on the orientations of the nodes' spatial distributions:

- *homogeneous anisotropic.* In this case, the spatial distribution of the nodes are perfectly aligned, i.e., the direction associated to δ_1 (and δ_2) is the same for all nodes.
- *heterogeneous anisotropic.* In this case, the spatial distribution of the nodes are not aligned. This happens when the direction associated to δ_1 is selected at random for each node, according to any angle distribution (independent of n) different from the deterministic distribution.

Discussion: The anisotropic mobility case is interesting because it can describe the situation in which the mobility pattern of nodes does not equally fill a bidimensional space in all

[4]It is possible to combine together the multiple home-point extension with the anisotropic mobility extension, however we have preferred to analyze the two extensions separately.

TABLE I
SYSTEM PARAMETERS

Symbol	Meaning	Range
n	number of nodes	$\gg 0$
L_n	edge length of network area	≥ 1
α	scaling exponent of L_n: $L_n = n^\alpha$	$0 \leq \alpha \leq 1/2$
δ	decay exponent for isotropic restricted mobility	≥ 0
δ_1,δ_2	decay exponents for anisotropic restricted mobility	≥ 0
m	number of clusters	$1 < m \leq n$
ν	scaling exponent of m: $m = n^\nu$	$0 < \nu \leq 1$
r	radius of the disk centered at the cluster middle point in which home-points are uniformly located	$0 < r < d_C$
ζ	decay exponent of the distance between the two home-points of a node	≥ 0

TABLE II
SUMMARY OF NOTATION

Symbol	Meaning
$\mathcal{O}_n$	network area (torus surface)
X_i^h	location of the primary home-point of node i
Y_i^h	location of the secondary home-point of node i
$\phi(d)$	isotropic pdf of one node around its home-point
$E[d]$	average distance reached by a node from one home-point
$\phi(d_1, d_2)$	anisotropic pdf of one node around its home-point
ρ_C	cluster density over the network area
d_C	average distance between two cluster middle points
$E[z]$	average distance between the two home-points of a node
R_T	common radio range
λ	average rate of a single traffic flow
$h(n)$	per-node capacity (or per-node throughput)

directions, but it is more highly concentrated over uni-dimensional paths. Consider for example a urban context with human or vehicular mobility. In this case the mobility of nodes (especially the communications among them) mainly takes place on the roads and their intersections, whereas nodes can be considered to be static outside the streets. The case of a regular grid of roads can be described by our Clustered Grid Model coupled with an anisotropic mobility model with either δ_1 or δ_2 greater than 2 (so that the stationary distribution of each node is non-negligible only over a unidimensional domain).

The system parameters for the class of networks considered in our work are summarized in Table I.

B. Interference Model

We assume that interference among simultaneous transmissions is described by the *protocol model* introduced in [9], that roughly represents the behavior of CSMA wireless protocols (e.g., 802.11) in the case of omni-directional antennas[5]. According to this model, nodes employ a common range R_T for all their transmissions (equivalently, they employ a common power level, i.e., no power adaptation mechanism is used). Node i is allowed to transmit to node j at time t, only if: i) the distance between i and j is no more than R_T, i.e., $d_{ij}(t) < R_T$; ii) for every other node k simultaneously transmitting, $d_{kj}(t) > (1 + \Delta) R_T$, being Δ a guard factor. We assume that transmissions occur at fixed rate which is normalized to 1.

C. Traffic Model

Similarly to previous work we consider uniform permutation traffic matrices, i.e., traffic patterns in which n randomly selected source-destination pairs (s, d) exchange traffic at rate λ. Source-destination pairs are selected in such a way that every node is origin and destination of a single traffic flow with average rate λ. Formally, a uniform permutation traffic matrix is defined by $\mathbf{\Lambda} = \lambda[\lambda_{sd}]$, where $[\lambda_{sd}] \in \{0, 1\}$, $\sum_s \lambda_{sd} = 1$, $\forall s$, and $\sum_d \lambda_{sd} = 1$, $\forall d$.

We say that the *per-node capacity* (or maximum per-node throughput) of the system is $\Theta(h(n))$ if, given a sequence of uniform permutation traffic patterns with rate $\lambda^{(n)} = h(n)$, there exist two constants c, c' such that $c < c'$ and both the following properties hold:

$$\begin{cases} \lim_{n\to\infty} \Pr\{c\lambda^{(n)} \text{ is sustainable}\} = 1 \\ \lim_{n\to\infty} \Pr\{c'\lambda^{(n)} \text{ is sustainable}\} < 1 \end{cases}$$

Equivalently, we say in this case that the *network capacity* (or maximum network throughput) is $\Theta(nh(n))$.

A summary of the notation used throughout the paper is reported in Table II.

D. Preliminaries

In the following arguments regarding the random models, we will need this lemma, which has been widely used in previous work:

Lemma 1: Consider a set of n points $\mathbf{X}$ uniformly and independently distributed over a bidimensional domain $\mathcal{O}$ of surface L^2. Let $\mathcal{A}$ be a regular tessellation of $\mathcal{O}$ (or any sub-region of $\mathcal{O}$), whose tiles A_k have a surface $|A_k|$ not smaller than $16\frac{\log n}{L^2}$, $\forall k$. Let $U(A_k)$ be the number of points of $\mathbf{X}$ falling in A_k. Then, uniformly over the tessellation, $U(A_k)$ is comprised w.h.p. between $\frac{n|A_k|}{2L^2}$ and $\frac{2n|A_k|}{L^2}$, i.e., $\frac{n|A_k|}{2L^2} < \inf_k U(A_k) \leq \sup_k U(A_k) < \frac{2|A_k|n}{L^2}$.
We do not repeat the proof of this lemma, which is based on a standard application of the Chernoff bound (see for example [27]).

III. Paper contribution

In [22] we have identified two main regimes that arise within our family of mobile networks, under the restriction of a single home-point per node: the **super-critical** regime, which occurs when $\alpha < \nu/2$ (i.e., in cohesive, *dense* networks, where $d_C \to 0$), and the **sub-critical** regime, characterized by $\alpha > \nu/2$ (i.e., in *sparse* networks, where $d_C \to +\infty$).

We have then developed a general framework for the analysis of the capacity scaling properties in the super-critical regime. Our main finding, schematically represented in Fig. 2, is that the per-node capacity is $\Theta(n^{-\alpha})$ with high probability[6] (w.h.p.), *independently* of both the shape $\phi(d)$ of the spatial distribution around the home-point (provided that it has finite first and second moments) and of the parameter ν of the particular Clustered Random model (including the special case of $\nu = 1$, i.e., the Uniform model). The rational behind this behavior is the following: in the super-critical regime, every node, for effect of

[5]The protocol model has been proven to be pessimistic with respect to the physical model employing power control (see Theorem 4.1, pag. 174 in [26]). Thus the results obtained in this paper can be regarded as lower bounds of the network capacity achievable under the physical model employing power control.

[6]I.e., with a probability tending to 1 as $n \to \infty$

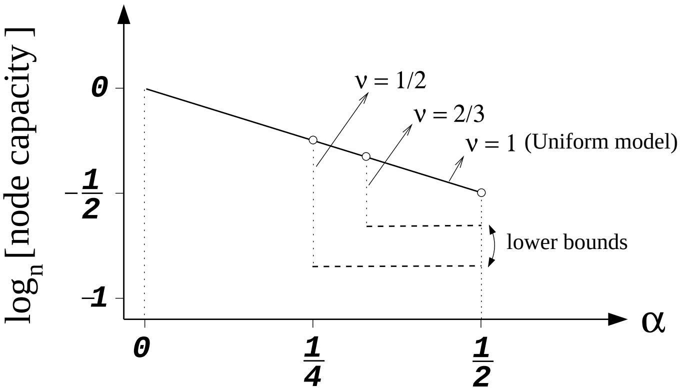

Fig. 2. Capacity results obtained in our previous work

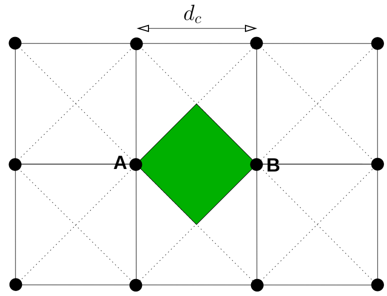

Fig. 3. Regular distribution of clusters over the area and scheduling region between clusters A and B (shaded area)

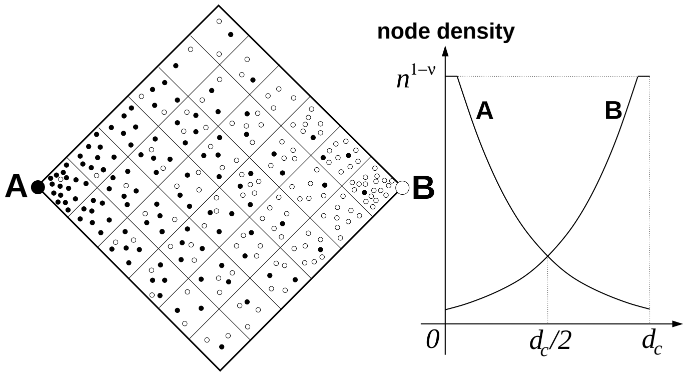

Fig. 4. Density of nodes belonging to clusters A and B in the scheduling region between them

mobility, spans an area $\Omega(d_C^2 \log n)$ around its own home-point (independently of the shape of $\phi(d)$). Thus, local effects due to clustering are washed out, because node pairs belonging to different clusters whose middle points are separated by a distance $O(d_C \sqrt{\log n})$ gets regularly in contact with each other and thus have the opportunity of directly exchanging data.

In [22] we have also made a preliminary investigation of the *sub-critical* regime ($\alpha > \nu/2$), limiting our attention to the case in which the node spatial distribution $\phi(d)$ has finite support. In this case, mobility no longer helps, since the area spanned by individual nodes around their home points becomes $o(d_C^2)$. Hence nodes belonging to different clusters never meet and the per-node capacity is abruptly lowered to a constant value $\Theta(n^{\nu/2-1}/\log n)$ (for any $\alpha > \nu/2$), equivalent to that of a static network.

The purpose of this paper is to complement and generalize results presented in [21], [22] along several directions: i) offering a complete panorama of system behavior in the sub-critical regime for general spatial distributions $\phi(d)$ (Section IV-A); ii) studying the case in which the average distance reached by a node from the home-point is not independent of n (in order sense), but scales itself with the network size, tending to infinity as the number of nodes (or, equivalently, the network physical extension) increases (Section IV-B); iii) extending the analysis to the interesting cases in which either a node has several home-points variably distributed in the area (Section V), or the spatial distribution of each node around its home point is not rotationally-invariant (Section VI).

In contrast to the *super-critical* regime, we will see that in the *sub-critical* regime the specific shape of $\phi(d)$ is important. In particular, the tail behavior of $\phi(d)$ plays a crucial role as it determines the frequency at which nodes belonging to different clusters get in contact with each other.

IV. The basic scenario

In this section we analyze network scenarios in which each node has a single home-point, or, more in general, multiple home-points having average inter-distance $E[z] = o(d_C)$. In Section IV-A we analyze the sub-critical regime in the case of general spatial distributions for which the average distance $E[d]$ reached by a node is finite ($\delta > 3$). In Section IV-B we extend the analysis to the case of infinite $E[d]$ ($\delta < 3$). A graphical representation of our results is reported and discussed in Section IV-C.

A. Analysis of the sub-critical regime for $\delta > 3$

We remind that the sub-critical regime is characterized by $\alpha > \nu/2$. In this case, different spatial distributions resulting from the node mobility process can impact the network capacity only in the presence of clustering ($\nu < 1$), otherwise for $\nu = 1$ the network behaves as a static one as soon as $\alpha = 1/2$ (see Fig. 2). As a preliminary step, we start considering the Clustered Grid model (Sec. IV-A.1), which is simpler to analyze. Then we show how our results can be extended to the Clustered Random model (Sec. IV-A.3).

1) The Clustered Grid model: In this case clusters' centers form a regular square grid, where the distance between neighboring clusters is $d_C = n^{\alpha-\nu/2}$. We now anticipate what is, in this case, the scheduling/routing scheme that achieves the maximum network capacity, and later show, in Section IV-A.2, that it is not possible to do any better than the proposed scheme.

A source, that wishes to send a packet to a destination belonging to a different cluster (almost all of the flows), routes it through a sequence of relay nodes corresponding to an horizontal and/or vertical sequence of adjacent clusters. We call such sequence a *logical* path. Given the next hop of a logical path, the actual communication takes place when the node gets sufficiently close to any relay node whose home-point belongs to the next hop cluster. Since the length of each logical hop is equal to d_C, the average number of hops to reach a destination is $\Theta(n^{\nu/2})$.

Now, let us consider an arbitrary pair of neighboring clusters A and B. Transmissions between two nodes whose home-points belong to A and B, respectively, are scheduled only in the region of points having either A or B as the closest cluster (shaded area in Fig. 3), so as to maximize the density of transmitter-receiver

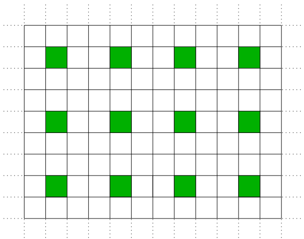

Fig. 5. Example of non-interfering subset of squarelets with $M = 9$.

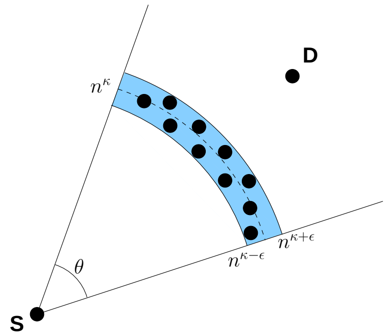

Fig. 6. Next-hop clusters that can be chosen by an alternative routing strategy that selects logical hops of length $\Theta(n^{\kappa})$

pairs in it. By so doing, we partition the network into $2n^{\nu}$ regions of area $\Theta(d_C^2) = \Theta(n^{2\alpha-\nu})$, each of them entirely dedicated to scheduling transmissions between nodes whose home-points belong to adjacent clusters. To determine the total number of transmissions that can be scheduled in the network according to the proposed scheme, we make use of the following lemma:

Lemma 2: Let $\sigma > \psi > 0$. In a region of area $\Theta(n^{\sigma})$, in which the minimum between the density of transmitters and receivers is, at any point, $\Theta(n^{-\psi})$, it is possible to schedule $\Theta(n^{\sigma-\psi})$ non-interfering transmissions.

Proof: (Lemma 2). We divide the region into a regular tessellation where each element has area n^{ψ}. Every element of the tessellation contains one transmitter-receiver pair with probability $\Theta(1)$, thus, by the law of large numbers, there are $\Theta(n^{\sigma-\psi})$ elements containing at least one transmitter-receiver pair. To avoid interference, it is sufficient to enable transmissions only in one out of $M = \Theta(1)$ elements of the tessellation, regularly spaced (for example, in the case of a square tessellation, we employ a maximum transmission range $R_T = \sqrt{2}n^{\psi/2}$, and we have $M = 9$, assuming a protocol interference model with $\Delta = 0$, see Fig. 5). ■

In our case, the behavior of the density of transmitters and receivers in a given region is illustrated in Fig. 4. The density of nodes whose home-points belong to cluster A (or B) behaves as $n^{1-\nu}/d^{\delta}$, where d is the distance from the cluster middle-point. It is easily seen that the maximum of the minimum density of transmitters and receivers is achieved in the middle of the region, for $d = d_C/2$, which scales in the same way as at $d = d_C$, i.e., at the extremes of the region. Hence, at any point, the minimum between the density of transmitters and receivers scales as $\Theta(n^{1-\nu}/d_C^{\delta}) = \Theta(n^{1-\nu-(\alpha-\nu/2)\delta})$. We can thus apply Lemma 2 with $\sigma = 2\alpha-\nu$, $\psi = -(1-\nu-(\alpha-\nu/2)\delta)$, obtaining that in each region we can schedule $n^{(\alpha-\nu/2)(2-\delta)+(1-\nu)}$ non-interfering transmissions. We observe that, in proximity of either A and B, there are fewer limitations caused by interference, because nodes coming from the other cluster get surrounded by a large number of nodes belonging to the local cluster, and thus one can use a transmission range small enough to prevent interference with nearby transmissions. However this does not change the scaling order of the number of transmissions that can occur simultaneously, which is any case limited by the minimum between the density of transmitters and receivers.

Notice that in our routing scheme network load is split evenly among the clusters, and since relays are exploited at random in each cluster, no node is overloaded. Since there are $\Theta(n^{\nu})$ regions, and the average number of hops to reach the destination is $n^{\nu/2}$, thus we can claim the following:

Proposition 1: The per-node capacity in the Clustered Grid model, according to the proposed scheduling-routing scheme, is

$$h(n) = \Theta\left(n^{(\alpha-\nu/2)(2-\delta)-\nu/2}\right) \quad (1)$$

2) A heuristic proof of optimality: In this section we provide an intuitive explanation to the fact that it is not possible to get higher capacity than that achieved by the above scheduling/routing scheme for the sub-critical regime. Notice that, for any value of δ, nodes occasionally get in contact with nodes belonging to far away clusters, however in the proposed scheme we *only* enable communication between nodes whose home-points belong to adjacent clusters (at distance d_C). In the following, we provide a heuristic argument to the claim that no capacity gain can be obtained by allowing transmissions between nodes belonging to clusters separated by distance $d = \omega(d_C)$, as long as $\delta > 3$. Therefore the scheme proposed in Section IV-A.1 is optimal (in order sense). In particular we will limit ourselves to consider clusters separated by a distance of the form n^{κ}. A formal proof of our result in the more general case is reported in Appendix I.

Let us consider an arbitrary flow between two nodes whose home-points belong to clusters S and D, respectively (see Fig. 6). The typical distance between S and D is of the order of n^{α}, the network physical extension. Suppose that we employ logical hops longer than $d_C = n^{\alpha-\nu/2}$. In particular, suppose we enable transmissions between nodes whose home-points belong to clusters separated by a distance $\Theta(n^{\kappa})$, with $\kappa > \alpha - \nu/2$. To include all of the possible logical next-hop at distance $\Theta(n^{\kappa})$, we actually consider all clusters located at distances n^{x}, with $\kappa - \epsilon < x < \kappa + \epsilon$, where ϵ is a small constant. In Fig. 6 we illustrate the first logical hop according to this alternative routing policy. Notice that we can choose any cluster in a sector of width $\theta < \pi$ (represented by the shaded area in Fig. 6) and arrive at the destination in a number of hops equal to $n^{\alpha-\kappa-\epsilon}$, under the optimistic hypothesis that at each hop the distance between two consecutive clusters is equal to $n^{\kappa+\epsilon}$.

Now, the area size of the considered sector is $\theta(n^{2(\kappa+\epsilon)} - n^{2(\kappa-\epsilon)}) = \Theta(n^{2(\kappa+\epsilon)})$. Therefore the number of clusters in which we can find next-hop nodes is $\Theta(\rho_C\ n^{2(\kappa+\epsilon)}) = \Theta(n^{\nu+2(\kappa+\epsilon-\alpha)})$. We can limit ourselves to the case in which transmissions between nodes belonging to a given cluster and their next-hops occur in a dedicated area of size $\Theta(d_C^2)$ around the cluster itself (at any point in between two clusters the minimum density of transmitters/receivers is of the same order of magnitude) and apply again Lemma 2, considering now the best case in terms of transmitter/receiver density, which is when the distance between two consecutive

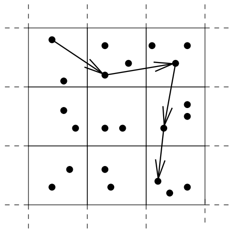

Fig. 7. Example of logical routing path across home-points in the Clustered Random model

clusters is always equal to $n^{\kappa-\epsilon}$. Applying the lemma with $\sigma = 2\alpha - \nu$, $\psi = -1 - 2(\kappa + \epsilon - \alpha) + (\kappa - \epsilon)\delta$, and considering that the number of hops according to the alternative scheme is $n^{\alpha-\kappa-\epsilon}$, we obtain the following upper bound to the per-node throughput, denoted by $h'(n)$,

$$h'(n) = O(n^{\kappa(3-\delta)+\epsilon(3+\delta)-\alpha}) \quad (2)$$

which has to be compared with (1). Now for any $\delta > 3$ and any $\kappa > \alpha - \nu/2$, choosing $\epsilon < (\kappa - \alpha + \nu/2)(\delta - 3)/(\delta + 3)$ it results $h'(n) = o(h(n))$. We remark that, by construction, $h'(n)$ provides a tight estimate of the throughput only when ϵ becomes negligible with respect to κ; thus even if for large values of ϵ, we can obtain $h'(n) = \omega(h(n))$, this does not imply that the throughput provided by the alternative scheme exceeds $h(n)$, as shown in Appendix I.

Therefore, exploiting logical hops of any length $d = \omega(d_C)$ provides no gain in terms of network capacity: even in the optimistic situation in which we can schedule such transmissions *in addition* to those of the previous scheme, the dominant contribution to network capacity is still given by logical hops of length $\Theta(d_C)$.

3) Extension to the Clustered Random model: When clusters are placed uniformly at random in the area, we partition the network into a regular square tessellation whose elements have area $(16 + \delta)n^{2\alpha-\nu} \log n^{\nu} = \Theta(n^{2\alpha-\nu} \log n)$. Lemma 1 guarantees that, with high probability, the number of clusters falling in each squarelet of the considered tessellation can be replaced by its expected value, which is $\Theta(\log n)$. We adopt again a routing strategy in which data are sent along a logical path formed by a sequence of relay nodes whose home-points belong to adjacent squarelets. A relay node is selected at random in each squarelet. A simple routing scheme (see Fig. 7) according to which data first move horizontally in a row of adjacent squarelets till reaching the column of the destination, and then vertically up to the target node, guarantees that traffic is distributed uniformly in the network and no squarelet is overloaded. The average number of hops of the proposed routing scheme is $\Theta(n^{\nu/2}/\sqrt{\log n})$. The scheduling scheme enables transmissions between a node and its next hop along the route at any point of the two adjacent squarelets, irrespective of the actual locations of the home-points of the nodes within the squarelets. The number of transmissions that can be enabled in each squarelet can be computed using the same rationale of Lemma 2. Since the typical distance between clusters belonging to adjacent squarelets is $d_H = \Theta(n^{\alpha-\nu/2}\sqrt{\log n})$, the minimum density of transmitters/receivers at any point is of the order of $n^{1-\nu}/d_H^{\delta}$, and thus in each squarelet one can schedule $\Theta\left(n^{(\alpha-\nu/2)(2-\delta)+1-\nu}(\log n)^{1-\delta/2}\right)$ transmissions. Considering that the number of hops is $\Theta(n^{\nu/2}/\sqrt{\log n})$, we obtain:

Proposition 2: The per-node capacity in the Clustered Random model is

$$h(n) = \Theta\left(n^{(\alpha-\nu/2)(2-\delta)-\nu/2}(\log n)^{\frac{1-\delta}{2}}\right) \quad (3)$$

which differs from the one computed in the case of a regular grid of clusters (1) by the factor $(\log n)^{\frac{1-\delta}{2}}$.

B. The case $\delta < 3$

When $\delta < 3$, the average distance reached by a node from its home-point is no longer finite, but scales with n, as described in Section II-A. Moreover, in this case Equation (2) suggests that long logical hops are better off than short logical hops. Actually, according to (2), to maximize the capacity one should exploit maximum-length logical hops (denoted as "longest hops") to reach the destination (i.e., maximize κ), thus delivering data over routes comprising only a finite number of hops. The simplest scheduling/routing scheme, in this regime, is the well known 2-hops relay scheme proposed by Grossglauser-Tse. Notice that, when δ goes below the critical value of 3, the optimal routing scheme shifts abruptly from using the "shortest hops" to using the longest hops, without intermediate regimes.

We now evaluate how the network capacity scales in the case of $\delta < 3$. Essentially, we need to determine how many transmissions can be scheduled between nodes whose home-points are separated by a distance n^{α} (almost all flows require at least one logical hop of length $\Theta(L_n)$, the physical network extension). Using the same arguments as before, we can limit ourselves to a scheduling policy in which transmissions of nodes belonging to a given cluster are enabled only in the region of area $\Theta(n^{2\alpha-\nu})$ around the cluster itself. Notice that there are $\Theta(n^{\nu})$ clusters that can be exploited as next-hop to reach a destination at the typical distance n^{α}.

We can thus apply again Lemma 2 with $\sigma = 2\alpha - \nu$, $\psi = -1 + \alpha\delta + \log G(n)$, where $G(n)$ is the normalization constant of the spatial distribution of a node, which can scale with n as well (Section II-A). We claim:

Proposition 3: The per-node capacity in both Clustered Grid and Clustered Random models when $\delta < 3$ is given by

$$h(n) = \Theta\left(\frac{n^{\alpha(2-\delta)}}{G(n)}\right) \quad (4)$$

Now, for $2 < \delta < 3$, $G(n)$ is finite, thus the per-node capacity scales as $\Theta(n^{\alpha(2-\delta)})$. For $0 < \delta < 2$, we have $G(n) = \Theta(L_n^{2-\delta})$, and we obtain a per-node capacity $\Theta(1)$, the same as in the Grossglauser-Tse case. We should mention that for $0 < \delta < 2$ our scheduling scheme, according to which transmissions are enabled only in a scheduling region of size $\Theta(d_C^2)$ around the cluster of the transmitter, does not always guarantee a per-node capacity $\Theta(1)$. This because the probability that a transmitter is found within its own scheduling region behaves as $\Theta\left((d_C/L_n)^{(2-\delta)}\right)$, which is $o(1)$ for $\delta < 2$, as opposed to the case $\delta > 2$, where it is $\Theta(1)$. Therefore, for $\delta < 2$ it is better to allow transmissions anywhere in the network irrespective of the location of home-points, just as in the Grossglauser-Tse scheme.

C. Graphical representation of capacity results

A schematic representation of the capacity results derived in previous sections is illustrated in Fig. 8, for the case $\nu = 1/2$. For $\delta < 3$, the capacity is independent of ν, and therefore there are no super- or sub-critical regimes, but a single operational regime for all values of (α, ν). This behavior can be explained by the fact that in this case the best scheme employs transmissions between

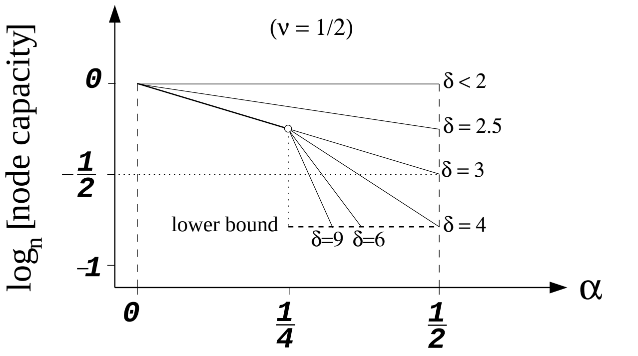

Fig. 8. Illustration of capacity results in the case of single home-point, for the particular value $\nu = 1/2$.

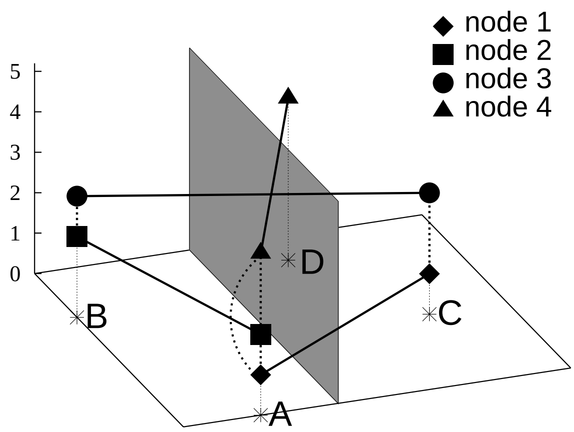

Fig. 9. Example of embedding of graph $G(V,E)$ into the 3D domain. Node 1 has home-points at clusters A and C; node 2 has home-points at clusters A and B; node 3 has home-points at clusters B and C; node 4 has home-points at clusters A and D; horizontal, solid edges have capacity 1; vertical dotted edges have capacity $n^{-\nu}$. The vertical plane provides a possible graph cut.

pairs of nodes whose home-points are far away from each other, hence local effects due to clustering are not important.

For $\delta > 3$, instead, the best scheme enables transmissions among pairs of nodes belonging to neighboring clusters. Since for $\alpha > \nu/2$ the typical distance d_C between neighboring clusters tends to infinity, transmission opportunities between node pairs are essentially related to the tail behavior of $\phi(d)$, hence become sensitive to δ. As δ increases, the likelihood of node contacts decreases, and the corresponding network capacity reduces. In the limit of $\delta \to \infty$, we approach the phase-transition behavior at $\alpha = \nu/2$ observed in the case in which $\phi(d)$ has finite support.

When $\delta = 3$ the capacity is the same as in the super-critical regime, i.e., $n^{-\alpha}$. When $\delta = 4$, at $\alpha = 1/2$ we get (for any ν) the same capacity as the lower bound of a static network, i.e., $n^{\nu/2-1}/\log n$. In general, if $\delta < 4$ (and $\alpha < 1/2$) the scheme proposed in Section IV-A for the sub-critical regime allows to achieve a better capacity than the one obtained in a static network. If $\delta > 4$, there is a value α^* after which mobility can not be anymore exploited to increase the network throughput. In such cases network capacity is achieved by a scheme that ignores mobility (i.e., according to which $R_T = \Theta(d_C)$). The critical value of α turns out to be $\alpha^* = \frac{2-4\nu+\nu\delta}{2(\delta-2)}$.

V. Two home-points

We now extend our analysis to the case in which every node has two home-points located at distinct clusters (i.e., at distance $\Omega(d_C)$). As we will see, in this case it is possible to achieve a capacity gain provided that the average distance between the primary and the secondary home-point of a node is larger (in order sense) than the average distance reached by a node around one home-point, i.e., $E[z] = \omega(E[d])$.

A. The Clustered Grid Model

We consider forwarding strategies according to which only intra-cluster transmissions are allowed, i.e., transmissions among nodes whose home-points belong to the same cluster. Notice that information can be effectively transferred from one cluster to another when nodes migrate from their primary to their secondary home-points. Indeed, the periodic migrations between the two home-points of a node permit to establish a "virtual tunnel" of capacity $\Theta(1)$, through which information can be conveyed. Moreover a communication link of capacity $\Theta(n^{-\nu})$ exists between every pair of nodes belonging to the same cluster [21], [22]. We observe that this strategy is somehow the opposite of the one proposed in Section IV for the single home-point case, where only transmissions between nodes belonging to different clusters are scheduled.

The system can be represented by a graph $G(V,E)$ with $|V| = 2n$, in which a couple of vertices v_i^1,v_i^2 is associated to each node i of the network. Graph $G(V,E)$ is embedded into the three dimensional domain $\mathcal{O}_n \times [1,n]$, being vertices v_i^1 and v_i^2 respectively placed at points (X_i^h, i) and (Y_i^h, i). An edge of unitary capacity joins v_i^1 and v_i^2. Edges of capacity $n^{-\nu}$ join pair of vertices belonging to the same cluster (i.e. with the same projection on $\mathcal{O}_n$). An example of such a graph in the case of 4 nodes is reported in Fig. 9.

We now compute an upper bound on the network transport capacity adopting a standard technique that resorts to the evaluation of the capacity of graph cut-sets (see Appendix II). In our case we consider the cut-set induced by a vertical plane (i.e., orthogonal to O_n) that divides the network area $\mathcal{O}_n$ in two congruent parts $\mathcal{A}$ and $\mathcal{B}$. We observe that the capacity crossing the considered plane, denoted by μ_{cross}, by construction equals the number of nodes having one home-point in $\mathcal{A}$ and one in $\mathcal{B}$; thus, μ_{cross} is a random variable, measurable over the σ-field defined by the location of the nodes' home-points; however, as direct consequence of the strong law of large numbers, when $n \to \infty$, w.h.p. this random variable takes a value equal, in order sense, to its mean (averaged over all configurations of home-points in the area).

The computation of $E[\mu_{\text{cross}}]$ can then be reduced to the evaluation of the probability that a node having its primary home-point in $\mathcal{A}$ has its secondary home-point in $\mathcal{B}$. We have:

$$E[\mu_{\text{cross}}] = 2n\Pr\{Y_i^h \in \mathcal{B}, X_i^h \in \mathcal{A}\} =$$
$$4\int_{\mathcal{A}} \Pr\{Y_i^h \in \mathcal{B} \mid X_i^h = (x,y) \in \mathcal{A}\}\rho_0 \,\mathrm{d}x\,\mathrm{d}y$$

where $\rho_0 = n^{1-2\alpha}$ is the average node density within $\mathcal{O}_n$. After some calculations it results:

$$\mu_{\text{cross}} = \begin{cases} \Theta(n) & \zeta < 1 \\ \Theta(n^{1+(1-\zeta)\nu/2}) & 1 < \zeta < 2 \\ \Theta(n^{1-\nu/2}) & \zeta > 2 \end{cases}$$

Dividing by the number of source/destination traffic relations whose source home-point lies in $\mathcal{A}$ and destination home-point in $\mathcal{B}$, which is $\Theta(n)$, we obtain the following upper bounds on the per-node capacity:

$$\hat{h}(n) = \begin{cases} \Theta(1) & \zeta < 1 \\ \Theta(n^{(1-\zeta)\nu/2}) & 1 < \zeta < 2 \\ \Theta(n^{-\nu/2}) & \zeta > 2 \end{cases} \tag{5}$$

No tighter bound can be found by selecting other surfaces and computing the capacity crossing the surface (for example cut-sets through horizontal planes are $\Theta(\min(n_+, n_-))$ by construction, being n_+ (n_-), the number of nodes placed above (below) the considered plane). Thus, exploiting the result in Appendix II, we obtain that a routing/forwarding scheme exists such that the per-node capacity is

$$h(n) = \Omega\Big(\frac{\hat{h}}{\log n}\Big)$$

Comparing the result in (5) with that in (4), we see that, when $E[d]$ scales with n ($\delta < 3$), the case of two home-points provides a higher capacity, in order sense, than in the case of a single home-point provided that $(1-\zeta)\nu/2 > \alpha(2-\delta)$. This is precisely the condition under which the average separation between primary and secondary home-points is larger than the average distance reached by a node around one home point (see expressions of $E[z]$ and $E[d]$ in Section II-A). For the same reason, since $E[d] = \Omega(1)$, we observe that the case of two-home-points is beneficial only when $E[z]$ does not tend to zero as n increases. In particular, for $\zeta \geq 2$, this implies that we must be in the case of $\alpha > \nu/2$, i.e., in the sub-critical regime.

Unfortunately, the theoretical additional capacity that is made available thanks to the presence of multiple home-points is more difficult to exploit than the capacity provided by node mobility around a single home-point. Indeed, finding the optimal routing/forwarding scheme requires, in general, the solution of a maximum concurrent flow problem over the contact graph of the nodes, which is enriched, in the case of multiple home-points, by random shortcuts. No simple throughput-optimal routing strategy can be found in this case, because the structure of the interconnection graph $G(V,E)$ does not permit, in general, to discover short sequences of chords connecting two assigned clusters, by employing only local geometrical considerations. This is related to the well known problem of navigating small-world graphs using only local connectivity information [28].

As an example, for $\zeta = 0$, the projection of graph $G(V,E)$ on $\mathcal{O}_n$, in which all nodes of $G(V,E)$ belonging to the same cluster collapse into a single node, can be regarded as an Erdös-Rényi[7] random graph $G(n,p)$ with $p = n^{-\nu}$. It is well known that on $G(n,p)$ no efficient routing strategies can be found employing only local information.

In the case of $\nu < 1/2$, for $\zeta < 1$ the projected graph degenerates w.h.p. into a fully connected mesh, where each pair of vertices is connected by $\Theta(n^{1-2\nu})$ parallel chords. In this particular case a simple four-hop optimal routing scheme can be defined to transfer data from any source to any destination, employing a "gateway" node that shares one home-point with the source node and one home-point with the destination node: data are conveyed from the source to the gateway (from the gateway to the destination node) employing a two-hops forwarding scheme within the source (destination) cluster. In this case the per-node throughput is exactly $\Theta(1)$.

We remark that allowing transmissions between nodes belonging to different clusters does not increase the overall capacity: even if inter-cluster transmissions could be scheduled *in addition* to intra-cluster ones, the dominant contribution would still be given by a scheme employing only intra-cluster transmissions, as can be easily verified comparing results obtained in the previous Section IV with those presented in this section.

The extension of the analysis to the Cluster Random and Uniform models are provided in Appendix III. Here we report the obtained results. The capacity under the Cluster Random model is equivalent to that of the Cluster Grid model. For the Uniform model, a small degradation of order $\log n$ occurs for $\alpha = 1/2$:

Proposition 4: In the case of the Uniform model, the per-node throughput is:

$$\hat{h}(n) = \begin{cases} \Theta(1) & \zeta < 1, \alpha < 1/2 \\ \Theta(1/\log n) & \zeta < 1, \alpha = 1/2 \\ \Theta(n^{(1-\zeta)/2}) & 1 < \zeta < 2 \\ \Theta(n^{-1/2}) & \zeta > 2 \end{cases}$$

VI. ANISOTROPIC MOBILITY

The detailed capacity analysis for anisotropic mobility is reported in Appendix IV. Here we summarize the main results for the homogeneous anisotropic mobility case:

Proposition 5: Assuming, without lack of generality, $\delta_1 > \delta_2$, the per-node throughput achievable under the Clustered Grid Model coupled with homogeneous anisotropic mobility is:

$$h(n) = \begin{cases} \Theta\left(n^{(\alpha-\nu/2)(1-\delta_1)-\nu/2}\right) & \text{for } \delta_1 > 2 \\ \Theta\left(n^{\alpha(1-\delta_1)}\right) & \text{for } 1 < \delta_1 < 2 \\ \Theta(1) & \text{for } 0 < \delta_1 < 1 \end{cases} \tag{6}$$

Proposition 6: Under the Clustered Random Model, the per-node throughput achievable under homogeneous anisotropic mobility is:

$$h(n) = \Theta\left(n^{(\alpha-\nu/2)(2-\delta_1-\delta_2)-\nu/2}(\log n)^{\frac{1-\delta_1-\delta_2}{2}}\right), \quad \text{for } \delta_1 > 2,\ \delta_2 > 1 \tag{7}$$

$$h(n) = \Theta\left(n^{(\alpha-\nu/2)(1-\delta_1)-\nu/2}(\log n)^{\frac{1-\delta_1}{2}}\right), \quad \text{for } \delta_1 > 2,\ \delta_2 < 1 \tag{8}$$

$$h(n) = \Theta\left(n^{\alpha(1-\delta_1)+(\alpha-\nu/2)(1-\delta_2)}(\log n)^{-\frac{\delta_2}{2}}\right), \quad \text{for } 1 < \delta_1 < 2,\ \delta_2 > 1 \tag{9}$$

$$h(n) = \Theta\left(n^{\alpha(1-\delta_1)}\right) \quad \text{for } 1 < \delta_1 < 2,\ \delta_2 < 1 \tag{10}$$

[7] Recall that Erdös-Rényi $G(n,p)$ graphs are w.h.p. connected whenever $p|V| > \log|V|$, i.e. $\nu < 1$

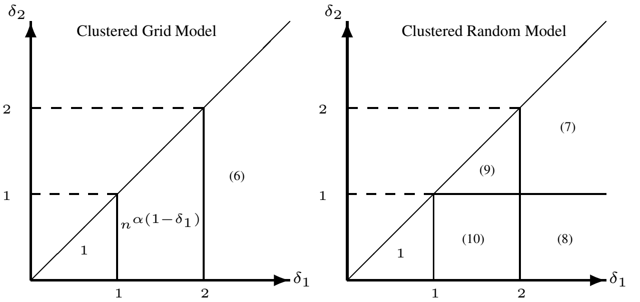

Fig. 10. Capacity results for anisotropic mobility: Clustered Grid Model (left diagram) and Clustered Random Model (right diagram). The equation corresponding to a particular region is specified in the figure.

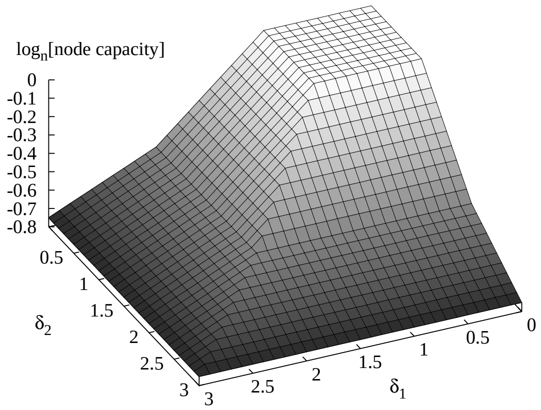

Fig. 11. Capacity in the case of Clustered Grid Model with $\alpha = \nu = 1/2$ (in log scale)

A. Summary of results for anisotropic mobility patterns

A summary of our capacity results for the anisotropic mobility pattern is reported in Fig. 10 for both the Clustered Grid and Clustered Random models. In Fig. 10, the subset of the plane (δ_1, δ_2) comprising all points for which $\delta_1 > \delta_2 > 0$ has been divided into regions characterized by the same scaling law (either reported explicitly or indicated within brackets). Notice that the behavior at the points for which $\delta_2 > \delta_1 > 0$ is symmetric with respect to the bisector.

In Fig. 11 and Fig. 12 we have reported the resulting capacity (in log scale) for all combinations of (δ_1, δ_2), in the particular case of $\alpha = \nu = 1/2$, for the Clustered Grid and Clustered Random models, respectively. The minimum value of $n^{-0.75}$ corresponds to the lower bound $n^{\nu/2-1}$ of a static network. We observe that, for a given tuple (δ_1, δ_2), the capacity under the Clustered Grid model is in general higher that the corresponding capacity under the Clustered Random model.

VII. Conclusion

In this paper, we have considered clustered, sparse networks of heterogeneous mobile nodes, in which the shape of the spatial distribution of each node around one or more home-points plays a fundamental role in determining the overall transport capacity. Complementing the results in [22], we have offered a complete view of the system behavior in the sub-critical regime, for general spatial distributions. We have also analyzed the case in which the average distance reached by a node from a home-point is not independent of n (in order sense), but scales itself with the network size. At last, we have extended the analysis to the interesting cases in which either a node has two or more home-points variably distributed over the network area or spatial distributions which are not rotationally-invariant around the home-point.

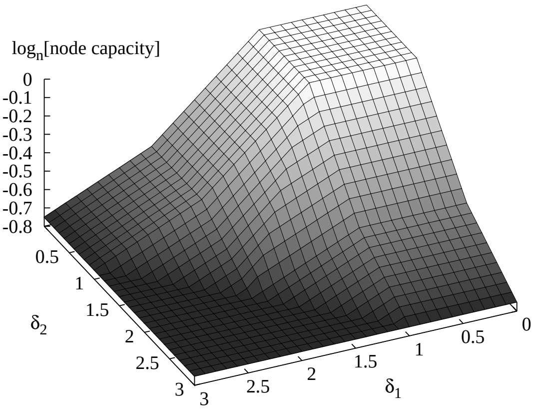

Fig. 12. Capacity in the case of Clustered Random Model with $\alpha = \nu = 1/2$ (in log scale)

Appendix I
Proof of optimality

An abstract representation of the network is provided by a geometric capacitated di-graph $G(V, E, \mu)$, (i.e. a directed graph in which a non-null capacity μ_{ij} is associated to every edge (i, j)). Vertex $i \in V$ corresponding to node i is placed at home-point X_i^h. Quantities μ_{ij} represent the long term mean rate at which data are sent from node i to node j according to the adopted scheduling/routing strategy.

Information flow $s \to d$ can follow different paths in $G(V, E, \mu)$ from s to d, each path corresponding to a different sequence of intermediate relay nodes.

Proposition 7: Defined $d_{ij}^h = \|X_i^h - X_j^h\|$, if traffic $\mathbf{\Lambda}$ is sustainable, then:

$$\lambda \sum_{sd} \lambda_{sd} d_{sd}^h \leq \sum_{ij} \mu_{ij} d_{ij}^h$$

Proof: The proof descends from the continuity constraints on routing. Indeed for any source-destination pair (s, d), traffic λ_{sd} must be routed in graph $G(V, E, \mu)$ through paths connecting s and d. The length of these paths, by triangular inequality, cannot be shorter than d_{sd}^h, thus the assert follows. ■

We introduce the cumulative quantity

$$T(d) = -\sum_i \sum_j \mu_{ij} \mathbb{I}_{d_{ij}^h > d}$$

representing the average aggregate rate exchanged among all nodes i and j, such that $d_{ij}^h > d$. From its definition $T(d + \Delta) - T(d) = \sum_i \sum_j \mu_{ij} \mathbb{I}_{d < d_{ij}^h < d+\Delta}$, and thus

$$\sum_i \sum_j \mu_{ij} d_{ij}^h = \int x \, \mathrm{d}T(x)$$

As a consequence, if we are able to lower bound $T(d)$ with a function $S(d)$, with $S(L_n) = 0$ (i.e., to find $S(d)$ such that $T(d) \geq S(d)$, $\forall d$), then we are able to upper bound the network throughput. Indeed, by integration per parts it can be easily shown that:

$$\int x\,\mathrm{d}T(x) \leq \int x\,\mathrm{d}S(x)$$

which implies:

$$\lambda \sum_{sd} \lambda_{sd} d^h_{sd} \leq \int x\,\mathrm{d}S(x)$$

In our case, for sufficiently large n, being $d^h_{sd} = \Theta(n^\alpha)$, w.h.p. it results:

$$\lambda = O\Big(\frac{1}{n^{\alpha+1}} \int x\,\mathrm{d}S(x)\Big)$$

To obtain an expression for $S(d)$, consider two mobiles i and j whose home-points are X^h_i and X^h_j, and let $\|X^h_i - X^h_j\| = d_{ij}$; the event $\{d_{ij}(t) < R_T\}$, by construction

$$\big\{d_{ij}(t) < R_T\big\} \subseteq \left\{\|X_i(t) - X^h_i\| > \frac{d - R_T}{2}\right\} \cup \left\{\|X_j(t) - X^h_j\| > \frac{d - R_T}{2}\right\}$$

i.e.,

$$\Pr\{d_{ij}(t) < R_T\} \leq \Pr\left\{\|X_i(t) - X^h_i\| > \frac{d - R_T}{2}\right\} + \Pr\left\{\|X_j(t) - X^h_j\| > \frac{d - R_T}{2}\right\} = 2\Pr\left\{\|X_i(t) - X^h_i\| > \frac{d - R_T}{2}\right\} = 2\int_{\mathcal{O}_n} \phi(x) \mathbb{I}_{x > \frac{d-R_T}{2}}\,\mathrm{d}x$$

Indeed, the aggregate number of transmissions $T(d)$ that can potentially occur between nodes whose home-points are separated by a distance greater than or equal to d, satisfies:

$$T(d) \geq -\sum_i E[\mathbb{I}_{\|X_i(t) - X^h_i\| > \frac{d-R_T}{2}}]$$

since for any such transmission at least one of the two nodes must be at a distance greater than or equal to $\frac{d-R_T}{2}$ from its home-point. Therefore,

$$T(d) \geq -2n \int_{\mathcal{O}_n} \phi(x) \mathbb{I}_{x > \frac{d-R_T}{2}}\,\mathrm{d}x = S(d)$$

Now, considering that the minimum distance between nodes belonging to different clusters is $d = d_C = n^{\alpha - \nu/2}$, and that $R_T = o(d)$, we obtain as upper-bound to the per-node capacity [8]

$$\lambda = O(n^{(\alpha - \nu/2)(2-\delta) - \nu/2})$$

which proves the optimality (in order sense) of the scheduling/routing scheme described in Section IV-A for $\delta > 3$.

The same reasoning can be employed to prove the optimality of the scheduling/routing scheme for the case of $\delta < 3$, and for the Clustered Random model (neglecting polylog terms).

[8] Indeed, notice that i) transmission between nodes residing in the same cluster are not effective to make information advance along its path $s \to d$ in graph $G(V, E, \mu)$; ii) if one selects $R_T = \Theta(d)$ the system performance gets severely degraded for effect of interference, resulting in the same per-node capacity as that of a static network, i.e., $\Theta(n^{\nu/2-1})$

Appendix II
Graphs cut-sets and network capacity

Consider a communication network admitting a representation in terms of an associated capacitated di-graph $G(V, E, \mu)$, i.e., a directed graph in which a non-null capacity μ_{ij} is associated to every edge (i, j).

The maximum transport capacity of the network under traffic matrix $\mathbf{\Lambda} = \lambda[\lambda_{sd}]$, can be formalized in terms of a Maximum Concurrent Flow (MCF) problem [29] over $G(V, E, \mu)$, i.e., in terms of the following multi-commodity flow problem over $G(V, E, \mu)$.

Having defined for every flow (s, d), and every $(i, j) \in E$ variables $f^{sd}_{ij} \in [0, 1]$, representing the average fraction of traffic from node s to node d, which is routed through edge (i, j), find:

$$\max \lambda$$

subject to the constraints:

$$\lambda \sum_s \sum_d \lambda_{sd} f^{sd}_{ij} \leq \mu_{ij}$$

$$\sum_i f^{sd}_{ij} - \sum_k f^{sd}_{jk} = \begin{cases} 1 & \text{for } j = d \\ 0 & \text{for } j \neq d \text{ and } j \neq s \\ -1 & \text{for } j = s \end{cases} \tag{11}$$

The set $\{f^{sd}_{ij}\}$ univocally defines the corresponding routing strategy in the network/graph.

Unfortunately, MCF problems are, in general, hard to solve; an upper bound to λ can be obtained in terms of graph cuts:

Proposition 8: Traffic $\lambda\mathbf{\Lambda}$ is sustainable only if, for any partition $(\mathcal{S}, \mathcal{D})$ of the nodes, it results:

$$\lambda \sum_{s \in \mathcal{S}} \sum_{d \in \mathcal{D}} \lambda_{sd} \leq \sum_{i \in \mathcal{S}} \sum_{j \in \mathcal{D}} \mu^n_{ij} \tag{12}$$

It has been proven in [29] that, in undirected graphs, traffic is guaranteed to be sustainable if the ratio between the minimum value of the r.h.s. and the maximum value of the l.h.s. in (12) is $\Omega(\log k)$, being k the number of flows.

Appendix III
Multiple home-points: extensions to Cluster Random and Uniform models

The extension of the analysis to the Cluster Random model is trivial when $r = \Theta(1)$, i.e., when the physical span of clusters does not scale with n. Indeed, in this case, even if nodes belonging to the same cluster do not share anymore the same home-point, they frequently come in contact for effect of mobility. A virtual communication link of capacity $\Theta(n^{-\nu})$ is still established between every pair of nodes belonging to the same cluster, by the adoption of simple scheduling policies. Thus the scheme described in the previous section allows to achieve the same capacity results.

When $r = \omega(1)$, contacts among nodes belonging to the same cluster become more sporadic, thus potentially determining a degradation of system performance; this because capacities among nodes belonging to the same cluster are not anymore homogeneously equal (in order sense) to $\Theta(n^{-\nu})$. From an analysis of the system, however, it emerges that, for any $\nu < 1$, the intra-cluster transport capacity never becomes the network bottleneck (i.e., cut-sets induced by horizontal planes of $G(V, E)$

do not provide tighter limits to the network capacity with respect to vertical planes).

For the Uniform model ($\nu = 1$), instead, a small throughput degradation of order $\log n$ occurs in the case $\zeta < 1$ and $\alpha = 1/2$. To summarize, we have:

$$\hat{h}(n) = \begin{cases} \Theta(1) & \zeta < 1, \alpha < 1/2 \\ \Theta(1/\log n) & \zeta < 1, \alpha = 1/2 \\ \Theta(n^{(1-\zeta)/2}) & 1 < \zeta < 2 \\ \Theta(n^{-1/2}) & \zeta > 2 \end{cases}$$

In the Uniform model, a scheme that obtains optimal throughput can be devised by dividing the domain $\mathcal{O}_n$ into squarelets of size 1×1 (for $\alpha < 1/2$) or $4\sqrt{\log n} \times 4\sqrt{\log n}$ (for $\alpha = 1/2$), and allowing transmissions only between nodes whose home-points reside in the same squarelet (in practice, nodes whose home-points fall within the same squarelet can be thought to belong to a virtual cluster equivalent to those of the Random Cluster Model). When $\alpha < 1/2$, $\Theta(n^{1-2\alpha})$ nodes fall within each squarelet; in addition, being the squarelets of unitary surface, nodes whose home-points belong to the same squarelet come frequently in contact; as a consequence a virtual communication link of capacity $\Theta(1/n^{1-2\alpha})$ exists between every pair of nodes within the same squarelet (to apply Lemma 2, a transmission range $R_T = n^{\alpha-1/2}$ should be selected in this case). When $\alpha = 1/2$, the number of nodes within each squarelet is $\Theta(\log n)$; in this case, a transmission range $R_T = \sqrt{\log n}$ must be selected. This because there are nodes in the network whose home-points have minimum distance $\Omega(\log n)$ from any other node home-point; as a result, the capacity of the virtual communication link between node pairs within the same squarelet becomes $\Theta(1/\log^2 n)$ for effect of interference among transmissions.

APPENDIX IV
ANISOTROPIC MOBILITY

We will first consider the Clustered Grid model, which is simpler to analyze thanks to the deterministic position of the clusters. The extension to the Clustered Random Model will be presented in Section IV-B.

A. The Clustered Grid Model

We start considering the homogeneous anisotropic scenario. Without loss of generality, we assume $\delta_1 \geq \delta_2$. First of all we generalize Lemma 2 to compute the number of transmissions that can be simultaneously scheduled in each scheduling region. This generalization is needed because the minimum between the density of transmitters and receivers is no longer uniform (in order sense) within a given scheduling region.

Lemma 3: In a region $\mathcal{A}$ of area $\Theta(n^\sigma)$, in which the minimum between the density of transmitters and receivers at point (x, y) is described by a measurable function $g(x, y)$, it is possible to schedule a number of transmissions $\Theta\left(\iint_{\mathcal{A}} g(x, y)\,\mathrm{d}x\,\mathrm{d}y\right)$, provided that $\iint_{\mathcal{A}} g(x, y)\,\mathrm{d}x\,\mathrm{d}y = \omega(1)$.

Proof: As a first step, the assertion can be easily proven when $g(x, y)$ is a simple measurable function (i.e., when $\mathcal{A}$ can be partitioned into a finite set of regular domains over which $g(x, y)$ is constant). Indeed, in this case it is sufficient to apply the arguments of Lemma 2 to each of the domains in which $g(x, y)$ is constant. Then the result can be extended to any general measurable function $g(x, y)$ applying standard Lebesgue integration techniques (i.e. evaluating the number of simultaneous transmissions for any simple function $\hat{g}_s(x, y)$ that approximate $g(x, y)$ from below, and then taking the supremum of these numbers.) ■

We start considering either the case in which $\delta_1 \geq \delta_2 > 2$ or the case in which $\delta_1 > 2 > \delta_2 > 1$. Repeating the arguments of Section IV-A it is easy to prove the throughput optimality of the scheduling/routing scheme proposed in Section IV-A according to which a source wishing to send a packet to a destination belonging to a different cluster, routes it through random relay nodes whose home-points belong to an horizontal and/or vertical sequence of adjacent clusters. While evaluating the number of transmissions that can be simultaneously scheduled in each region $\mathcal{S}$ of area $\Theta(d_C^2)$ as depicted in Fig. 3, an asymmetry arises between the areas belonging to horizontal paths and the areas belonging to vertical paths. The bottleneck is represented by the direction associated to δ_1 (let it be the x axis of a cartesian coordinate system) on which fewer transmissions can be simultaneously scheduled because mobility is more restricted along the direction associated to δ_1, resulting into a smaller density of transmitter-receiver pairs. Therefore, we can limit ourselves to computing the transport capacity of the system along the x axis. Similar considerations apply also to the case $\delta_1 > 2 > \delta_2 > 1$.

Applying Lemma 3, the number of transmissions between nodes belonging to horizontal neighboring clusters A and B can be evaluated as (see Fig. 3):

$$n^{1-\nu} \int_{\mathcal{S}} \frac{\min(1, d_C^{-\delta_1})}{G_1(n)} \frac{\min(1, y^{-\delta_2})}{G_2(n)}\,\mathrm{d}x\,\mathrm{d}y \approx$$
$$\approx n^{1-\nu} \int_0^{d_C} \frac{\min(1, d_C^{-\delta_1})}{G_1(n)}\,\mathrm{d}x \int_{-d_C/2}^{d_C/2} \frac{\min(1, y^{-\delta_2})}{G_2(n)}\,\mathrm{d}y =$$
$$= \Theta\left(n^{(\alpha-\nu/2)(1-\delta_1)+1-\nu}\right) \quad (13)$$

Taking into account that there are $\Theta(n^\nu)$ scheduling regions in the network and that packets have to be retransmitted $\Theta(n^{\nu/2})$ times along the horizontal path, the achievable per-node throughput results to be:

$$h(n) = \Theta\left(n^{(\alpha-\nu/2)(1-\delta_1)-\nu/2}\right), \quad \text{for} \quad \delta_1 > 2,\ \delta_2 > 1$$

When $\delta_1 > 2$ and $\delta_2 < 1$, instead, the previously described scheduling/routing scheme becomes penalizing in terms of throughput. A better throughput is achieved by a scheduling/routing scheme that allows transmissions between arbitrary pairs nodes belonging to clusters lying on adjacent columns (i.e., whose horizontal distance is d_C). This because the density of nodes at every point is dominated by the contribution provided by nodes whose home-points have vertical distance of order $\Theta(L_n)$. Notice that, according to this scheme, once the packet reaches a node whose home-point lies within the same column of the destination, the packet is directly forwarded to the destination, since for $\delta_2 < 1$ long hops along the vertical axis are more convenient than short hops.

Also in this case, the computation of the number of schedulable transmissions with horizontal drift (i.e. transmissions employing nodes residing in adjacent columns) can be carried out exploiting Lemma 3. The number of schedulable transmissions in each region $\mathcal{S}$ is given by:

$$n^{1-\nu} \int_0^{d_C} \frac{\min(1, d_C^{-\delta_1})}{G_1(n)}\,\mathrm{d}x \int_{-d_C/2}^{d_C/2} n^{\nu/2} \frac{L_n^{-\delta_2}}{G_2(n)}\,\mathrm{d}y =$$

$$= \Theta\left(n^{(\alpha-\nu/2)(1-\delta_1)+1-\nu}\right)$$

which is exactly the same as that obtained in (13), since also in this case the integral over y is $\Theta(1)$. As a consequence, the per-node capacity of the scheme proposed for $\delta_1 > 2$ and $\delta_2 < 1$ is the same as that reported in (6).

Next, we consider the case of $\delta_1 < 2$. Here the optimal routing scheme selects, in each scheduling region, receivers belonging to clusters separated by an horizontal distance $\Theta(L_n)$. The only difference with respect to the previous derivation is that now the integral over x provides the term

$$\int_0^{d_C} \frac{L_n^{-\delta_1}}{G_1(n)} n^{\nu/2}\,\mathrm{d}x = \frac{n^{\alpha(1-\delta_1)}}{G_1(n)}$$

while the number of hops to be traversed along the horizontal direction is $\Theta(1)$. Following the same reasoning as above, the factor resulting from the integral over y is $\Theta(1)$ for any δ_2, by either selecting receivers whose home-points have vertical distance d_C (for $\delta_2 > 1$), or receivers whose home-points have vertical distance $\Theta(L_n)$ (for $\delta_2 < 1$). Plugging in the expression for $G_1(n)$, the final per-node capacity turns out to be $h(n) = n^{\alpha(1-\delta_1)}$ for $1 < \delta_1 < 2$, and $h(n) = \Theta(1)$ for $0 < \delta_1 < 1$.

We now consider the Clustered Grid Model coupled with heterogeneous anisotropic mobility. It is rather immediate to show that the capacity achievable under an heterogeneous anisotropic scenario characterized by tuple (δ_1', δ_2') is, in order sense, equivalent to the the one achievable under an homogeneous anisotropic scenario having $\delta_1 = \delta_2 = \min(\delta_1', \delta_2')$. This because the two directions characterizing the mobility pattern of a node have random orientation, thus on any given direction (including the bottleneck one) we can get a finite fraction of the transport capacity associated to the minimum between δ_1' and δ_2'. We conclude that the analysis of the heterogeneous anisotropic case can be reduced to that of the homogeneous anisotropic case.

Remark 1. Notice that, for $\delta_1 > 2$, the above findings hold only in the sub-critical regime, i.e., when $\alpha > \nu/2$. The super-critical regime ($\alpha < \nu/2$) can be analyzed applying the methodology developed in [22]. The resulting per-node throughput in super-critical regime for $\delta_1 > 2$, is $h(n) = n^{-\alpha}$ independently of the shape of $\phi(d_1, d_2)$.

Remark 2. For $\delta_1 > 2$, the above findings are meaningful only when the capacity resulting from our scheduling-routing schemes is better that the lower bound achievable in a static network, i.e., $n^{\nu/2-1}/\log n$. Otherwise, it is always possible to devise a scheme which does not exploit mobility at all, and by just increasing the transmission range achieves the same capacity as that of a static network.

B. The Clustered Random Model

We now extend the analysis to the Clustered Random Model. As we will see, in this case the network capacity resulting from an anisotropic mobility pattern of parameters δ_1 and δ_2 can be significantly smaller with respect to the corresponding capacity achievable under the Clustered Grid Model. This because clusters' middle points are no longer constrained to lie on a regular grid, which can limit the system performance in the case in which neighboring clusters provide the dominant contribution to the density of transmitter-receiver pairs.

Without loss of generality, consider an homogeneous anisotropic model having $\delta_1 \geq \delta_2$, and let δ_1 be associated to the horizontal axis x. For $\delta_1 > 2$ and $\delta_2 > 1$, we propose the same scheduling/routing scheme of Section IV-A.3 according to which the network is partitioned into a regular square tessellation whose elements have area $(16 + \delta)n^{2\alpha-\nu}\log n^{\nu} = \Theta(n^{2\alpha-\nu}\log n)$, and transmissions along horizontal/vertical paths are allowed only between nodes whose home-points lie in adjacent squarelets. For simplicity, we will assume that squarelets have a square shape of edge $\Theta(n^{\alpha-\nu/2}\sqrt{\log n})$, although this choice can be slightly sub-optimal (by a poly-log factor) in some cases[9].

The number of transmissions that can be enabled in each squarelet can be again computed applying Lemma 3. Since in this case the typical distance between transmitter's and receiver's home-point in both directions is $d_x = d_y = \Theta(d_C\sqrt{\log n})$, the minimum density of transmitters/receivers at any point of the squarelet is of order $n^{1-\nu}d_x^{-\delta_1}d_y^{-\delta_2}$, thus in each squarelet one can schedule $\Theta\left(n^{(\alpha-\nu/2)(2-\delta_1-\delta_2)+1-\nu}(\log n)^{(2-\delta_1-\delta_2)/2}\right)$ transmissions. Considering that there are $n^{\nu}/\log n$ squarelets, and $n^{\nu/2}/\sqrt{\log n}$ hops to traverse, the final per-node capacity turns out to be:

$$h(n) = \Theta\left(n^{(\alpha-\nu/2)(2-\delta_1-\delta_2)-\nu/2}(\log n)^{\frac{1-\delta_1-\delta_2}{2}}\right), \quad \text{for } \delta_1 > 2,\ \delta_2 > 1 \qquad (14)$$

which differs by a factor $n^{(\alpha-\nu/2)(1-\delta_2)}(\log n)^{\frac{1-\delta_1-\delta_2}{2}}$ from the corresponding per-node throughput (6) obtained in the Clustered Grid model.

For $\delta_1 > 2$ and $\delta_2 < 1$, the penalty due to the misalignment of clusters along the y axis disappears thanks to the fact that it becomes convenient to select receivers whose home-points have vertical distance of order $\Theta(L_n)$, cancelling local effects. However, a throughput reduction with respect to the Clustered Grid model is still expected because the typical distance along the x axis between the home-points of transmitter and receiver becomes $d_C\sqrt{\log n}$ in the Clustered Random model. Considering again that there are $n^{\nu}/\log n$ squarelet, and $n^{\nu/2}/\sqrt{\log n}$ hops to go through, we get a throughput reduction[9] by a factor $(\log n)^{(1-\delta_1)/2}$:

$$h(n) = \Theta\left(n^{(\alpha-\nu/2)(1-\delta_1)-\nu/2}(\log n)^{\frac{1-\delta_1}{2}}\right), \quad \text{for } \delta_1 > 2,\ \delta_2 < 1$$

In the case of $1 < \delta_1 < 2$, transmissions along the x axis employ long logical hops, thus cancelling the effects due to the randomness of the horizontal distance between neighboring clusters. However we still need to distinguish two cases depending on the value of δ_2, since the density of transmitters/receivers within a squarelet depends on it. For $\delta_2 > 1$, we get again a throughput reduction of order $(d_C\sqrt{\log n})^{1-\delta_2}$ due to the randomness of the vertical distance between neighboring clusters. Considering that there are $n^{\nu}/\log n$ squarelets, and $\Theta(1)$ hops to go through, we get in this case[9]:

$$h(n) = \Theta\left(n^{\alpha(1-\delta_1)+(\alpha-\nu/2)(1-\delta_2)}(\log n)^{-\frac{\delta_2}{2}}\right), \quad \text{for } 1 < \delta_1 < 2,\ \delta_2 > 1$$

At last, for $1 < \delta_1 < 2$ and $\delta_2 < 1$, all local effects due to the randomness of the cluster locations vanish because the home-

[9] When $\delta_1 \neq \delta_2$, it is possible to achieve a poly-log gain with respect to the results reported in this work by partitioning the network into squarelets having a rectangular shape of edges d_C and $d_C \log n$.

points of any transmitter-receiver pair have both horizontal and vertical distance of order $\Theta(L_n)$:

$$h(n) = \Theta\left(n^{\alpha(1-\delta_1)}\right) \quad \text{for } 1 < \delta_1 < 2,\ \delta_2 < 1$$

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